4.1 The concept of vector

Definition (vector):

A **vector** is a directed line segment which has *magnitude, direction and orientation*. Examples of vectors include force, time, distance, velocity, acceleration…

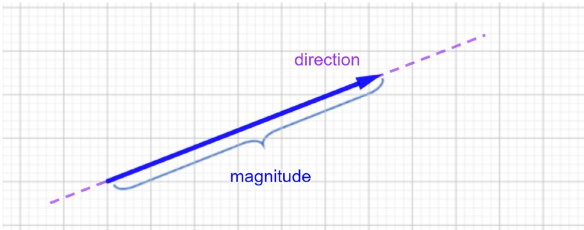

Figure 4.1 A vector

Vector vs Scalar

Numbers are called **scalars**. A scalar is a quantity which has magnitude only, but it does not any direction.

Notation

We denote vectors by labels of their initial and terminal points letters including arrows above the label, such as $\overrightarrow{AB}$ or small letters with arrows above the letter, such as $\vec{a}$.

The notation $\overrightarrow{AB}$ is useful because it indicates the orientation and location of the vector.

A Geometric View of Vectors

Geometrically, a vector or directed line segment is any ordered couple (A, B) of two arbitrary points A and B from space R^3, where A is the initial point, and B is the terminal point of this line segment.

A vector can be drawn as a directed line segment (as an **arrow**), whose length is the magnitude of the vector and with an arrow indicating the direction (orientation).

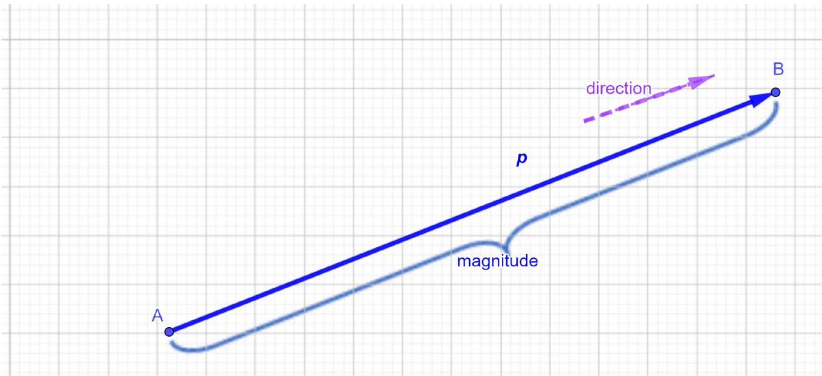

Figure 4.2 A vector is represented by a directed line segment from its initial point **A** to its

terminal point **B**

Line *p* on which the vector $\overrightarrow{AB}$ lies is called ***direction of this vector***. The ***direction*** of the vector is from its initial point A to its terminal point B.

Definition (magnitude or norm):

The ***magnitude*** or ***norm*** of the vector is the distance (the length of the line) between the initial and terminal points of the vector $\overrightarrow{AB}$ and is denoted as $|\overrightarrow{AB}|$.

Thus, if the initial point of a vector $\overrightarrow{AB}$ is $A(x_A, y_A)$ and the terminal point is $B(x_B, y_B)$, then the magnitude of a vector can be found using **Pythagoras's theorem**:

$$|\overrightarrow{AB}| = \sqrt{(x_B - x_A)^2 + (y_B - y_A)^2}$$

IMPORTANT NOTE

A magnitude is always a positive number. It is a scalar.

Types of Vectors

$\overrightarrow{AA} = \vec{0}$ is called ***zero - vector*** and its norm is equal to 0. The zero-vector is the only vector without a direction, and by convention can be considered to have any direction convenient to the problem at hand.

Unit vector of the vector $\vec{a}$ is the vector

$\vec{a}_0 = \frac{\vec{a}}{|\vec{a}|}$, $\vec{a} \neq \vec{O}$ i $|\vec{a}_0| = 1$.

To every point in a plane there can be assigned a unique vector whose initial point is in the origin *O* and the terminal point is in the given point $P(x_P, y_P)$. This vector is called the ***position*** or ***radius*** vector and it may be represented as (x_P, y_P) .

Example:

In the following figure, point *A* has the position vector $\vec{u}$ and point *C* has the position vector $\vec{v}$.

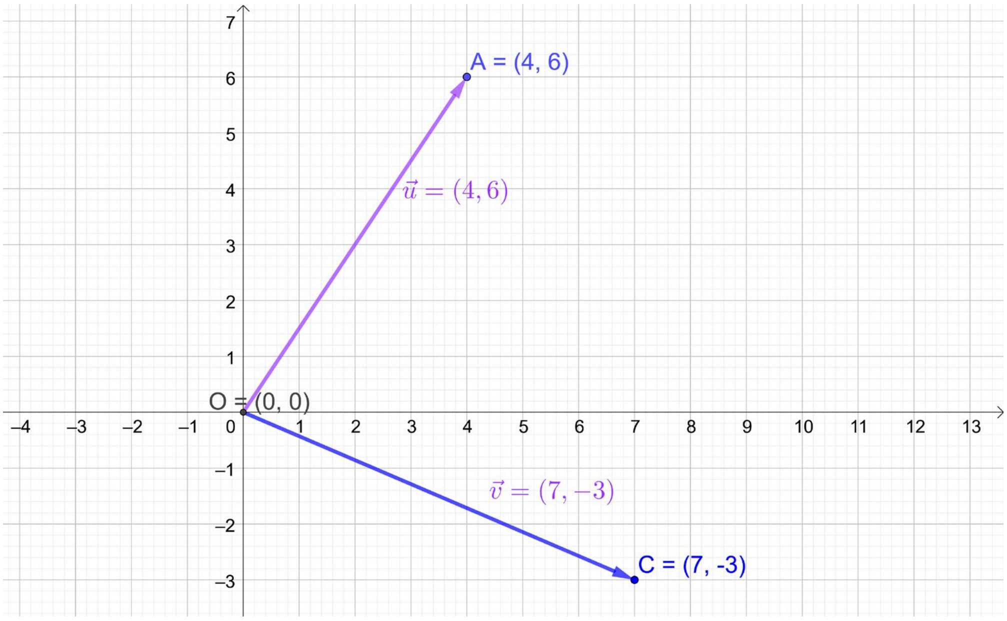

Figure 4.3

Vector $\overrightarrow{BA}$ is ***the opposite vector*** to vector $\overrightarrow{AB}$ and is designated $\overrightarrow{BA} = -\overrightarrow{AB}$.

The direction (orientation) of vectors

If two vectors $\overrightarrow{AB}$ and $\overrightarrow{CD}$ that lie on the same line or a parallel line to the same, then their direction (orientation) can only be equal or opposite. The direction of the vector is shown by the arrow at the end of the vector.

For two vectors $\overrightarrow{AB}$ and $\overrightarrow{CD}$ lying on the same line or the parallel lines, it holds:

1) they are of the same orientation if points *A* and *B* lie on the same side with regard to point *O*;

2) they are of the opposite orientation if the points *A* and *B* lie on different sides with regard to point *O*.

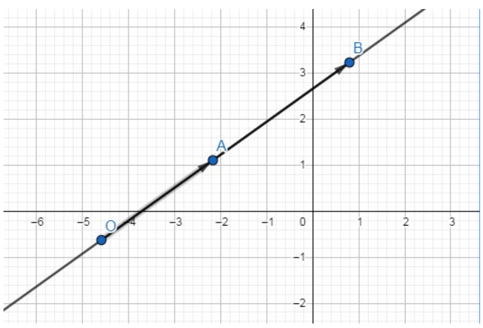

Figure 4.4 These position vectors $\overrightarrow{OA}$ and $\overrightarrow{OB}$ have the same orientation

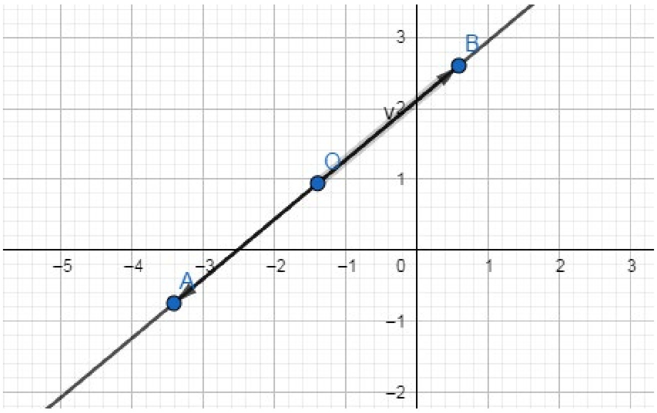

Figure 4.5 Vectors $\overrightarrow{\boldsymbol{OA}}$ and $\overrightarrow{\boldsymbol{OB}}$ are of the opposite orientation

Example

In **Figure 4.6** vectors $\overrightarrow{AB}$ and $\overrightarrow{CD}$ are parallel and have the same orientation.

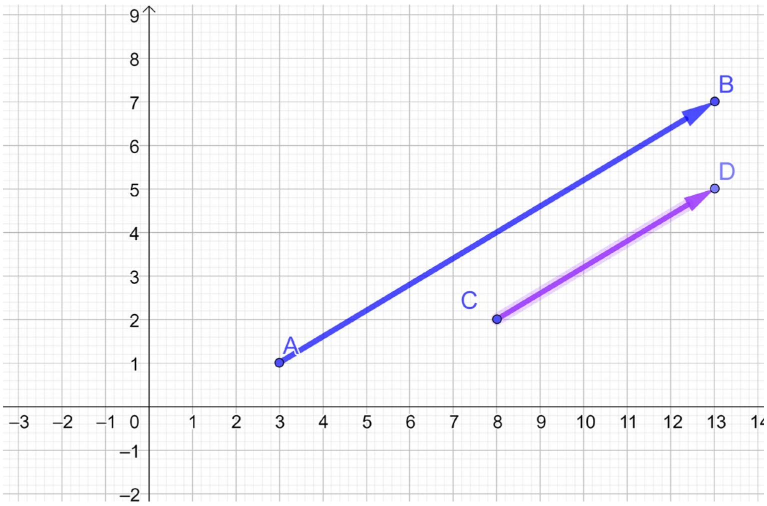

Figure 4.6 The vectors with the same orientation

In **Figure 4.7** vectors $\overrightarrow{AB}$ and $\overrightarrow{DC}$ are parallel and are going in the opposite orientation.

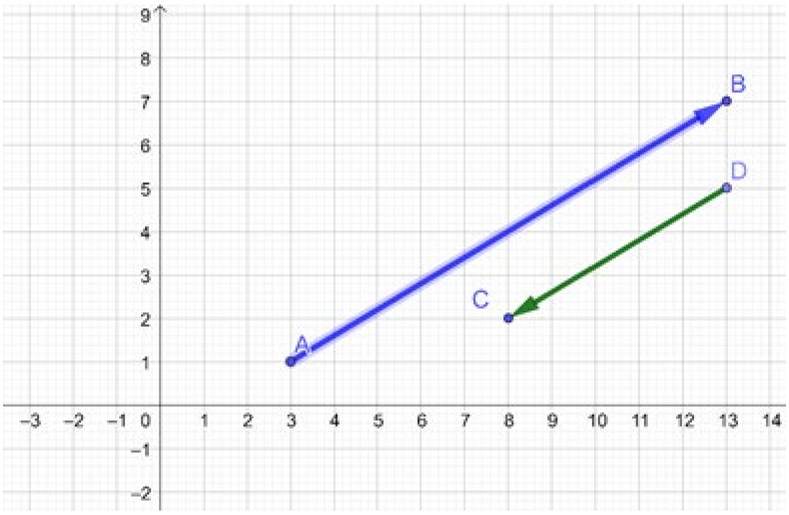

Figure 4.7 The vectors with the opposite orientation

Vectors $\vec{a}$ and $\vec{b}$ are mutually ***collinear*** if they lie in the same or in parallel lines, i.e. if there exists the number $\lambda \in$ R such that $\vec{\boldsymbol{a}} = \lambda\vec{\boldsymbol{b}}$. According to the agreement, zero - vector is ***collinear*** with any vector.

Example

All vectors on the figure below lie on the same line. The vectors are collinear vectors.

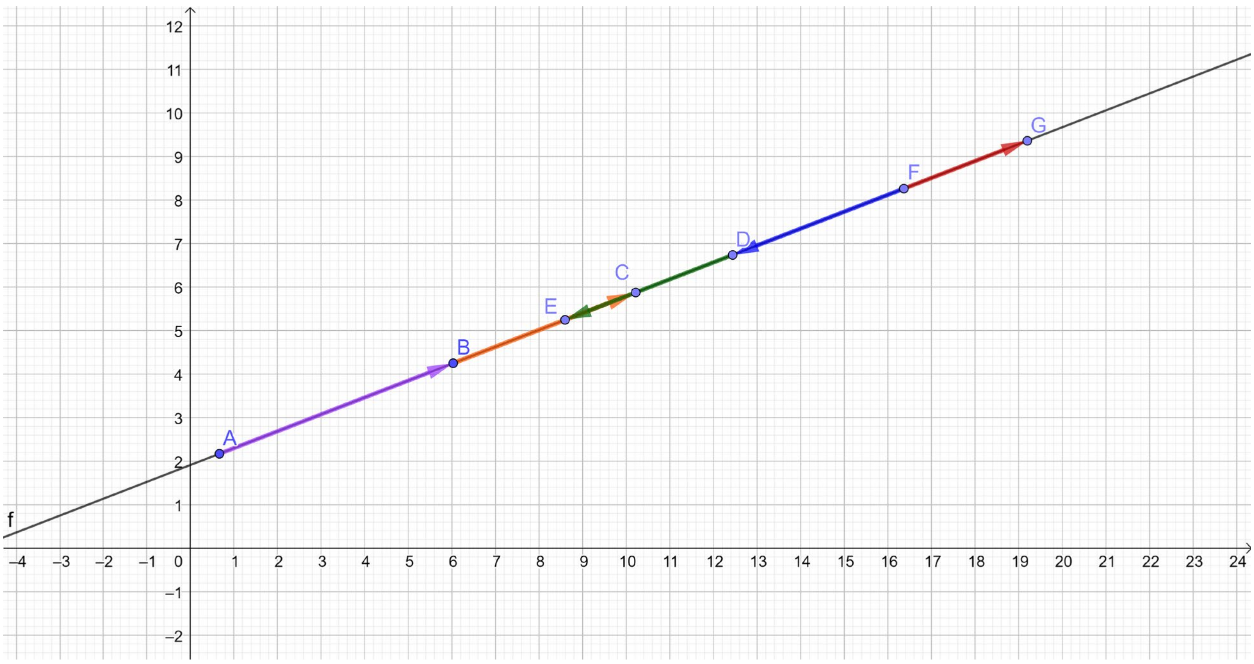

Figure 4.8 The example of collinear vectors

The following vectors on Figure 4.8 *with the same direction (orientation) are:*

- $\overrightarrow{AB}, \overrightarrow{BC}, \overrightarrow{FG}$
- $\overrightarrow{DE}, \overrightarrow{FD}$.

Vector coplanarity – a concept in geometry referring to the position of points and vectors. Four points in the space are ***coplanar*** if they lie in the same plane. Three vectors are ***coplanar*** (or linearly dependent) if they lie in a single plane (or parallel planes). Coplanar vectors $\vec{a}$, $\vec{b}$ and $\vec{c}$ may be represented as

$$\vec{c} = k\vec{a} + l\vec{b}, \qquad k.l \in \Re$$

Two vectors are ***equal*** and designated as $\vec{a} = \vec{b}$ if:

(1) they have the same length (magnitude)

(2) they are collinear vectors

(3) they have the same direction.

Example

Are the vectors $\vec{u} = \overrightarrow{AB}$ and $\vec{v} = \overrightarrow{CD}$ from the figure equal?

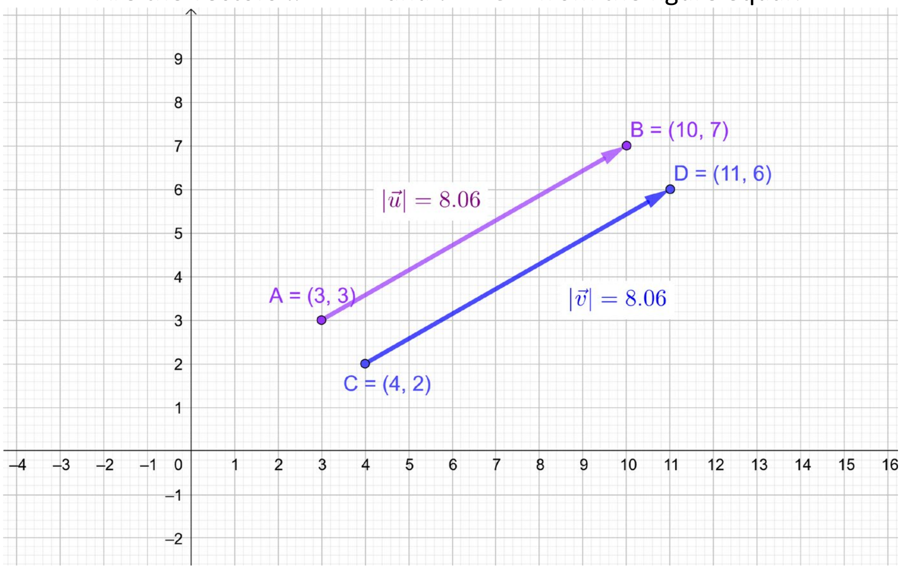

Figure 4.9

Solution:

Equal vectors have the same magnitude, the same direction and they are collinear vectors.

The vectors $\vec{u} = \overrightarrow{AB}$ and $\vec{v} = \overrightarrow{CD}$ **have the same magnitude**. The magnitude of the vector $\vec{u}$ is $|\vec{u}| = 8.06$ and of the vector $\vec{v}$ is $|\vec{v}| = 8.06$.

The vectors have the same directions and they lie in parallel lines.

Answer: The vectors $\vec{u} = \overrightarrow{AB}$ and $\vec{v} = \overrightarrow{CD}$ are equal vectors.

Exercices 4.1

For the isosceles trapezoid in **Figure 4.10** find:

a) Equal vectors

b) Opposite vectors
c) Collinear vectors

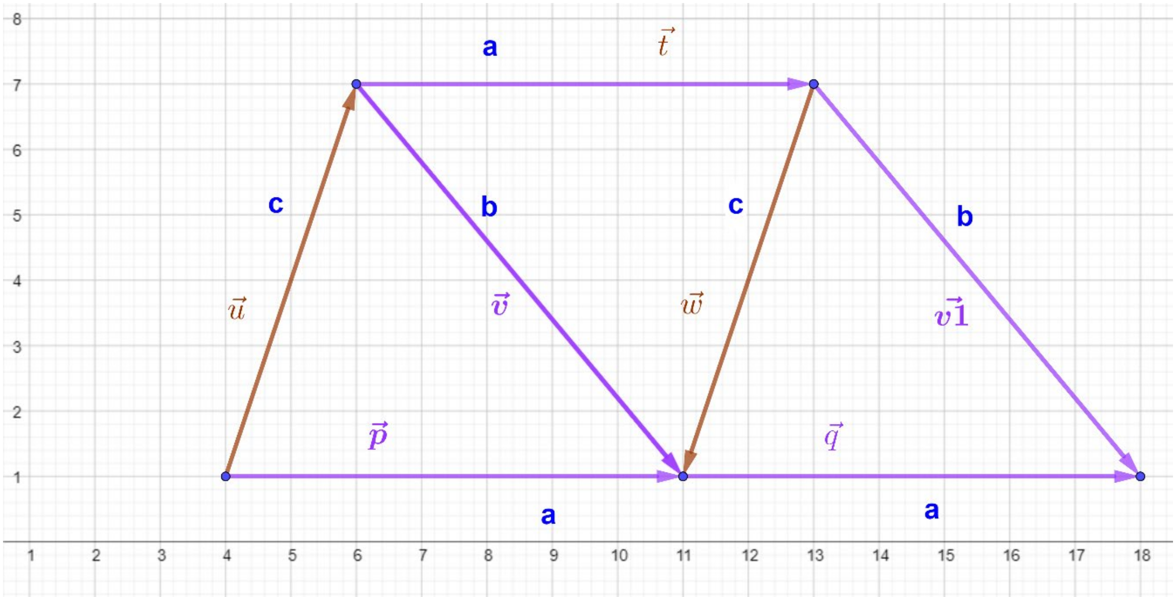

Figure 4.10

Solution:

d) Equal vectors: $\vec{p}=\vec{q}=\vec{t}$
e) Opposite vectors: $\vec{p}$ and $\vec{t}$, $\vec{q}$ and $\vec{t}$, $\vec{u}$ and $\vec{w}$, $\vec{v}$ and $\overrightarrow{v1}$
f) Collinear vectors: $\vec{p}, \vec{q}$ and $\vec{t}$, $\vec{u}$ and $\vec{w}$, $\vec{v}$ and $\overrightarrow{v1}$

Vectors $\vec{a}_1, \vec{a}_2, \ldots, \vec{a}_n$ are ***linearly independent*** if from the equation

$$\lambda_1\vec{a}_1 + \lambda_2\vec{a}_2 + \cdots + \lambda_n \overrightarrow{a_n} = \vec{0}$$ it follows that je $\lambda_1 = \lambda_2 = \cdots = \lambda_n = 0$.

Projection of vector $\vec{b}$ onto vector $\vec{a}$ is calculated according to the formula:

$proj_a \, \vec{b} = |\vec{b}| \cos \varphi$, where φ is the angle between vectors $\vec{a}$ and $\vec{b}$.