4.2 Three basic vector operations

Vector addition

The sum of two vectors $\vec{a}$ and $\vec{b}$ is the vector starting in the initial point of the first vector, and finishing in the final point of the second vector and is designated as $\vec{a} + \vec{b}$ (Figure 4.11). This approach is called the ***triangle method***.

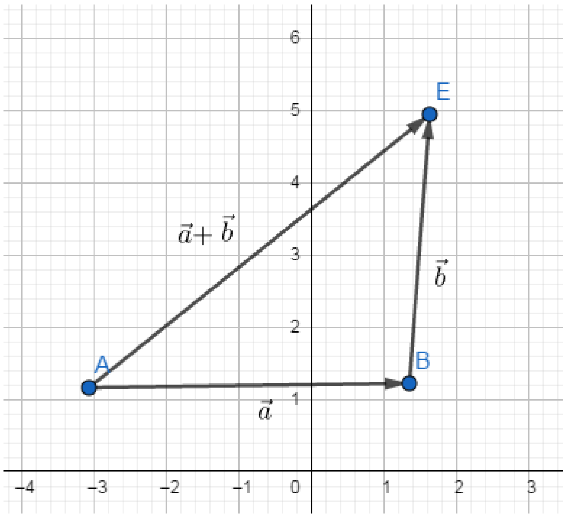

Figure 4.11 Adding vectors by the triangle method

The following features are valid:

(1) $(\vec{a} + \vec{b}) + \vec{c} = \vec{a} + (\vec{b} + \vec{c})$

(2) $\vec{a} + \vec{O} = \vec{O} + \vec{a} = \vec{a}$

(3) For each vector $\vec{a}$ there is the vector $\overrightarrow{a'}$ such that

$\vec{a} + \overrightarrow{a'} = \vec{a} + \overrightarrow{a'} = \vec{O}$.

(4) $\vec{a} + \vec{b} = \vec{b} + \vec{a}$

(5) $\|\vec{a} + \vec{b}\| \leq \|\vec{a}\| + \|\vec{b}\|$

Inequality (5) follows from the triangle method. These three vectors are sides of a triangle. Thus, it is true that the length of any one side is less than the sum of the lengths of remaining sides.

Vector subtraction

Subtracting a vector is the same as adding its negative.

The difference of the vectors $\vec{a}$ *and* $\vec{b}$ is the sum of $\vec{a}$ and $(-\vec{b})$.

We define:

$$\vec{a} - \vec{b} = \vec{a} + \left(-\vec{b}\right) = \vec{a} + (-1)\vec{b} \quad \text{or}$$

$$\overrightarrow{AB} - \overrightarrow{CD} = \overrightarrow{AB} + \left(-\overrightarrow{CD}\right) = \boldsymbol{\overrightarrow{AB}} + \boldsymbol{\overrightarrow{DC}}$$

The difference of two vectors is the vector starting in the initial point of the first vector, and finishing in the final point of the opposite vector of the second vector.

Example

Find $\vec{a} - \vec{b}$ for the vectors in **Figure 4.12**.

Solution:

<u>Step 1:</u> We reverse the direction of the vector $\vec{b}$ we want to subtract.

<u>Step 2:</u> Add the vector $\vec{a}$ and the opposite vector of the vector $\vec{b}$ as usual.

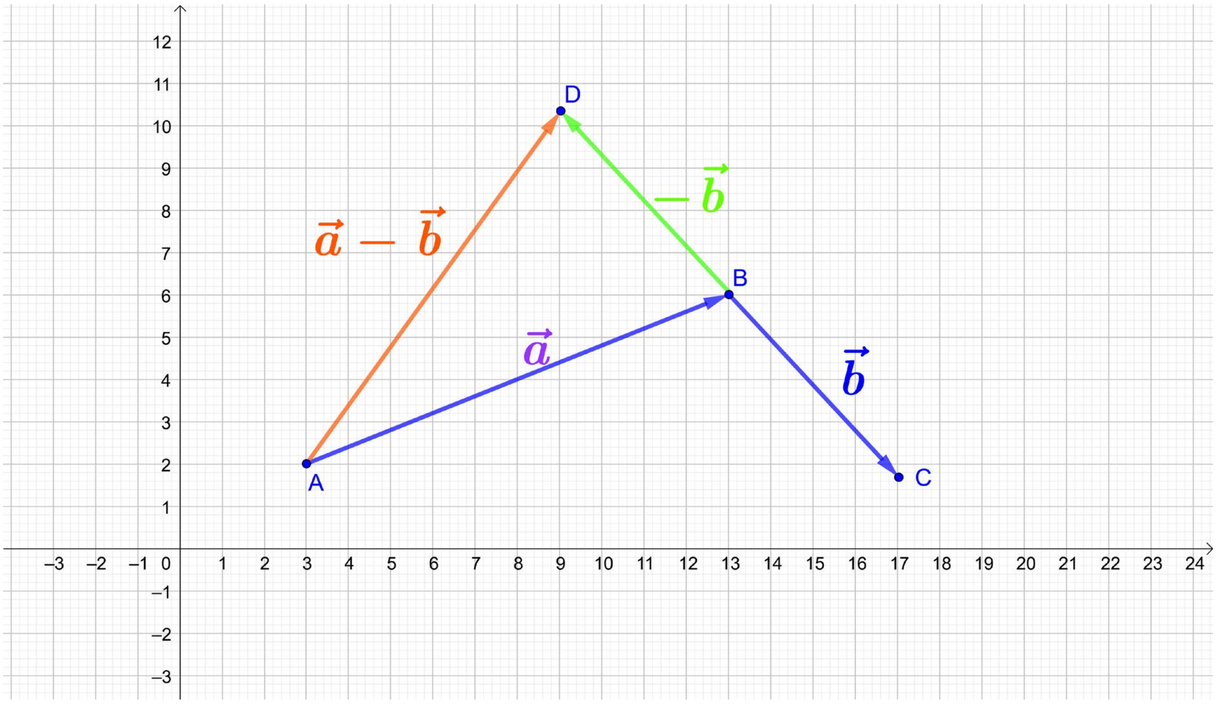

Figure 4.12 Vector subtraction

Multiplication of vector by a scalar (number)

With ***the multiplication by a scalar λ***, only the magnitude of the vector $\vec{a}$ is multiplied by the absolute value of the scalar λ. The result of the product $\lambda \cdot \vec{a}$ is the vector with a magnitude that is $|\lambda|$ times the magnitude of the vector $\vec{a}$ if $\lambda > 0$, and opposite direction if $\lambda < 0$.

The following applies:

1) vector norm $\lambda\vec{a}$ is equal to the product of absolute $|\lambda|$ and magnitude (norm) of $|\vec{a}\,|$;

$$|\lambda\vec{a}| = |\lambda||\vec{a}|.$$

2) vector $\lambda\vec{a}$ is collinear with vector $\vec{a}$.

The features of scalar multiplication by vector:

(1) $\lambda(\vec{a} + \vec{b}) = \lambda\vec{a} + \lambda\vec{b}$

(2) $(\lambda + \mu)\vec{a} = \lambda\vec{a} + \mu\vec{a}$

(3) $(\lambda\mu)\vec{a} = \lambda(\mu\vec{a})$

(4) $1 \cdot \vec{a} = \vec{a}$

(5) $(-1)\cdot\vec{a} = -\vec{a}$

Note that the vector $-\vec{a}$ has the same magnitude as $\vec{a}$, but has the opposite direction.