4.4 Vectors in rectangular coordinate system

Let *E* be the unit point on the *x*-axis, *F* the unit point on *y*-axis, *G* the unit point on *z*-axis and point *O* the origin in 3D-space R^3. Then radius vector $\overrightarrow{OE}$ is equal to unit vector $\vec{\imath}$, radius vector $\overrightarrow{OF}$ equal to unit vector $\vec{\jmath}$ and radius vector $\overrightarrow{OG}$ equal to unit vector $\vec{k}$.

The vectors $\vec{\imath}, \vec{\jmath}$ and $\vec{k}$ are the ***unit vectors*** in the positive ***x***, ***y***, and ***z*** direction, respectively. In terms of coordinates, we can write them as $\vec{\imath} = (1,0,0)$, $\vec{\jmath} = (0,1,0)$ and $\vec{k} = (0,0,1)$.

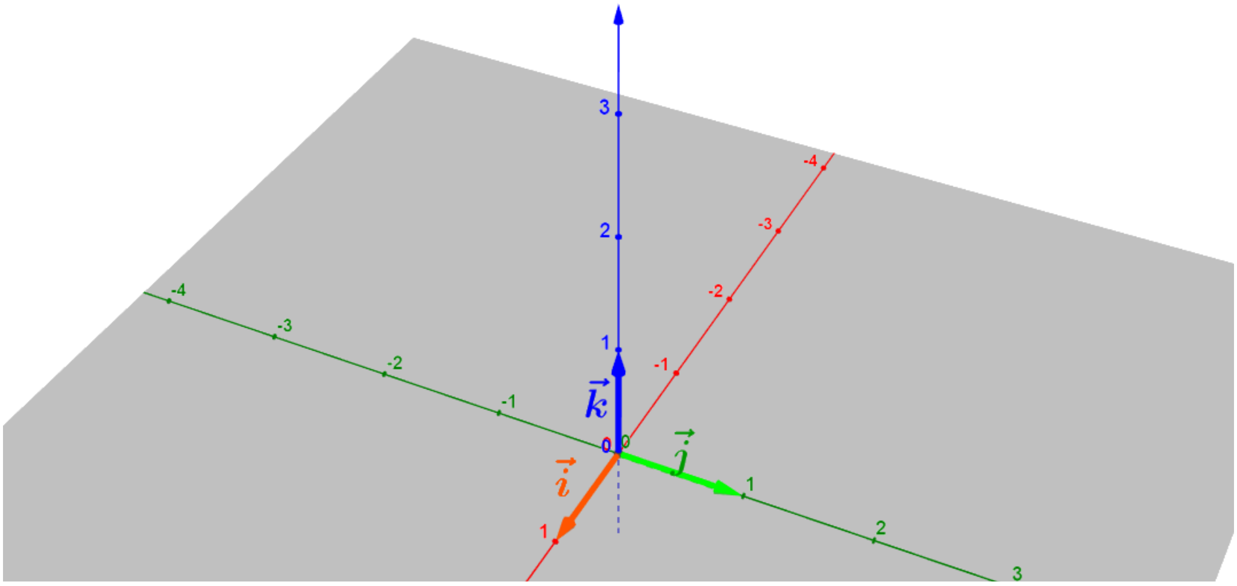


Figure 4.16 The standard unit vectors in three dimensions

A vector in three-dimensional space

Any point P in space can be assigned three coordinates $P = (a_x, a_y, a_z)$ and its position vector $\vec{a} = \overrightarrow{OP}$. In **Figure 4.17**, the vector $\vec{a}$ is drawn as the pink arrow with initial point fixed at the origin.

We assign coordinates of a vector $\vec{a}$ by orthogonal projecting the vector $\vec{a}$ on each axis x, y and z.

Black vectors $\overrightarrow{a_x} = \overrightarrow{OP_1}$, $\overrightarrow{a_y} = \overrightarrow{OP_2}$ and $\overrightarrow{a_z} = \overrightarrow{OP_3}$ show the projections of $\vec{a} = \overrightarrow{OP}$ on each axis and represent ***the scalar components or coordinates*** (a_x, a_y, a_z).

Any three-dimensional vector $\vec{a}$ can be represented as linear combination of three unit vectors $\vec{\imath}, \vec{\jmath}$, and $\vec{k}$ i.e. it can be expressed as the sum of the products of a scalar component and a unit vector lying on the corresponding coordinate axis in the form

$$\vec{a} = (a_x, a_y, a_z) = a_x\vec{\imath} + a_y\vec{\jmath} + a_z\vec{k}$$

The magnitude of that position vector of point P is equal to: $|\vec{a}| = \sqrt{(a_x)^2 + (a_y)^2 + (a_z)^2}$.

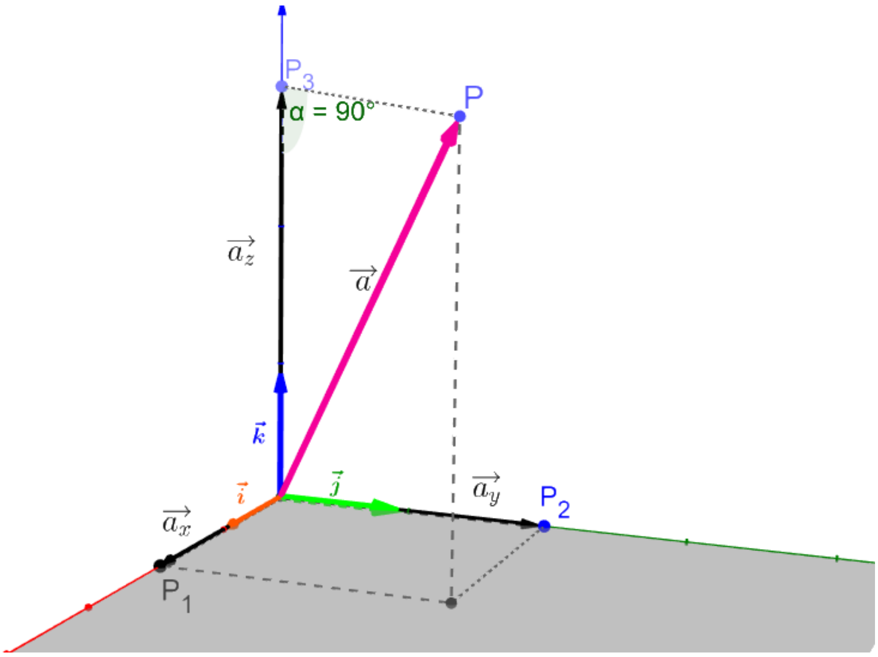


Figure 4.17 A vector $\vec{a}$ in three-dimensional space

Component Form of a Vector in three-dimensional space

Let be $\overrightarrow{AB}$ a vector with initial point $A(x_{i,}, y_{i,}, z_i)$ and terminal point $T(x_{t,}, y_{t,}, z_t)$. The component form of the vector $\overrightarrow{AB}$ can be expressed as $\overrightarrow{AB} = (x_{t,} - x_{i,},\ y_{t,} - y_{i,},\ z_{t,} - z_{i,})$.

The magnitude of that vector is equal to:

$$|\overrightarrow{AB}| = \sqrt{(x_{t,} - x_{i,})^2 + (y_{t,} - y_{i,})^2 + (z_{t,} - z_{i,})^2}.$$

A vector in the-plane

Each point P in the Cartesian system in the plane is identified with its *x* and *y* coordinates, $P(a_x, a_y)$.

Cartesian coordinates system in the plane is defined by an ordered triple $(O, \vec{\imath}, \vec{\jmath})$ where O is the origin, $\vec{\imath}$ and $\vec{\jmath}$ are two non-collinear unit vectors:

$\vec{\imath}$ - unit vector on the abscissa axis

$\vec{\jmath}$ - unit vector on the ordinate axis.

The position vector of the point P, $\overrightarrow{OP}$ may be represented as a linear combination of unit vectors:

$$\overrightarrow{OP} = a_x\vec{\imath} + a_y\vec{\jmath}$$

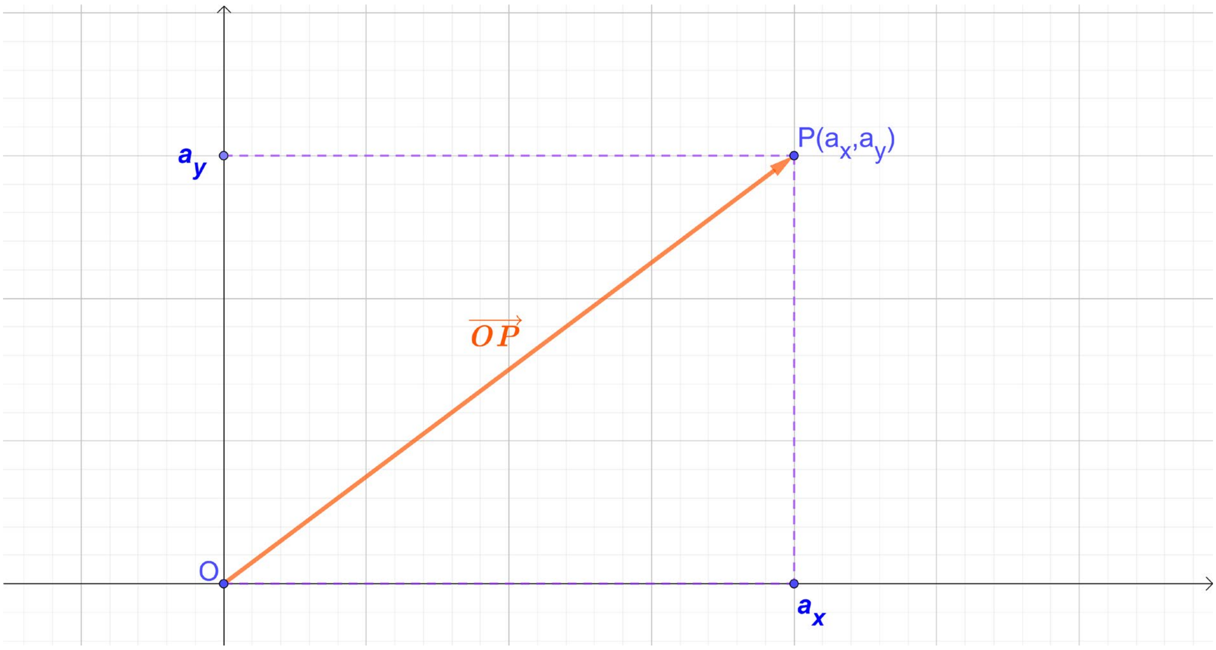


Figure 4.18 The components of a vector in the plane

Scalars a_x and a_y are called components of the vector $\overrightarrow{OP}$.

Using the Pythagorean Theorem, we can obtain an expression for the **magnitude** of a vector in terms of its components.

The magnitude of that position vector of point P is equal to:

$$|\overrightarrow{OP}| = \sqrt{(a_x)^2 + (a_y)^2}.$$

Component Form of a Vector in E^2

Let be $\vec{a}$ a vector with initial point $(x_{i,}, y_{i,})$ and terminal point $(x_{t,}, y_{t,})$. The component form of the vector $\vec{a}$ can be expressed as $\vec{a} = (x_{t,} - x_{i,},\ y_{t,} - y_{i,})$.

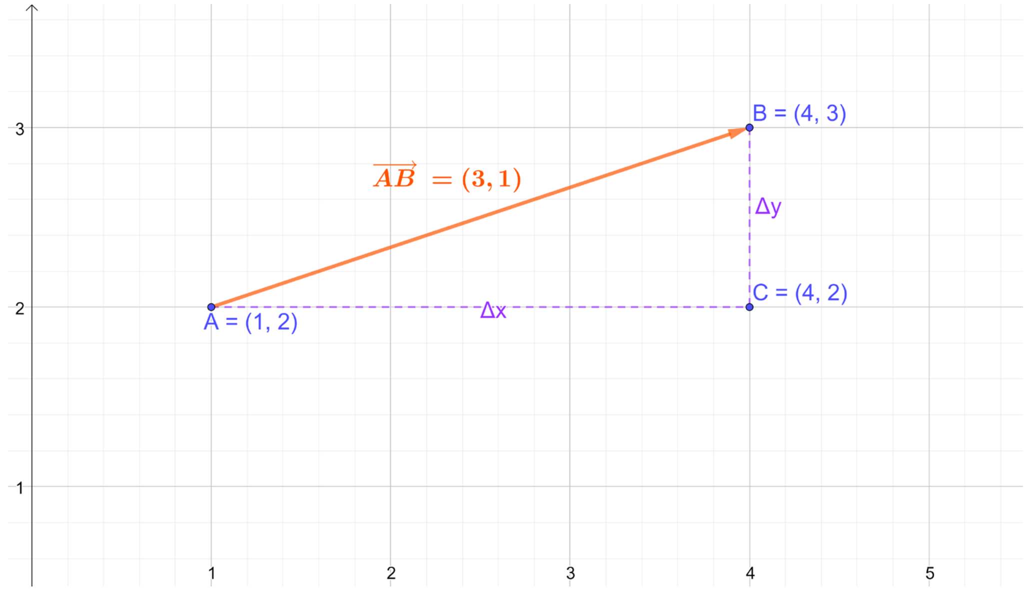
The magnitude of that vector is equal to:

$$|\vec{a}| = \sqrt{(x_{t,} - x_{i,})^2 + (\,y_{t,} - y_{i,})^2}.$$

Example:

Draw in the plane the vector $|\overrightarrow{AB}|$ whose initial point A is (1, 2) and terminal point B is (4, 3) and find its magnitude.

Solution:



$$|\overrightarrow{AB}| = \sqrt{(x_B - x_A)^2 + (y_B - y_A)^2}$$

$$|\overrightarrow{AB}| = \sqrt{(4-1)^2 + (3-2)^2}$$

$$|\overrightarrow{AB}| = \sqrt{3^2 + 1^2}$$

$$|\overrightarrow{AB}| = \sqrt{10} \approx 3.2$$

In some cases, only the magnitude and direction of a vector are known, not the points. For these vectors, we can identify the horizontal and vertical components using trigonometry (Figure 4.19).

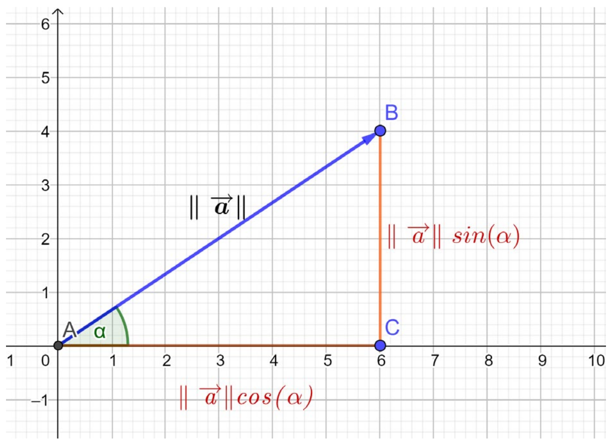


Figure 4.19 The components of a vector form the cathetus of a right triangle, with the vector as the hypotenuse