4.5 Performing Operations in Component Form

Scalar multiplication and Vector addition

- Scalar multiplication:

$$\lambda \cdot \vec{a} = (\lambda \cdot a_{x,}, \lambda \cdot a_{y,}, \lambda \cdot a_{z,})$$

- Vector addition:

$$\overrightarrow{a} + \overrightarrow{b} = (a_{x,}, a_{y,}, a_{z,}) + (b_{x,}, b_{y,}, b_{z,}) = (a_{x,} + b_{x,}, a_{y,} + b_{y,}, a_{z,} + b_{z,})$$

Example:

Let $\vec{a}$ be the vector with initial point (1, 1) and terminal point (3, -4) , and let $\vec{b} = (-1, 4)$.

a) Express $\vec{a}$ in component form and find $\|\vec{a}\|$. Then, using algebra, find
b) $\overrightarrow{a} + \overrightarrow{b}$
c) $3\,\overrightarrow{b}$
d) $\overrightarrow{2a} - \overrightarrow{b}$.

$Solution$:

a) $\overrightarrow{a}$ = (3-1, -4-1) =(2, -5)

$\overrightarrow{a} + \overrightarrow{b} = (2, -5) + (-1,4) = (2 + (-1), -5 + 4) = (1, -1)$ (orange vector on **Figure** 4.20)

b) $3\,\overrightarrow{b} = 3(-1,4) = (3 \cdot (-1), 3 \cdot 4) = (-3,12)$
c) $\overrightarrow{2a} - \overrightarrow{b} = 2 \cdot (2, -5) - (-1,4) = (4, -10) + (1, -4) = (4 + 1, -10 - 4) = (5, -14)$

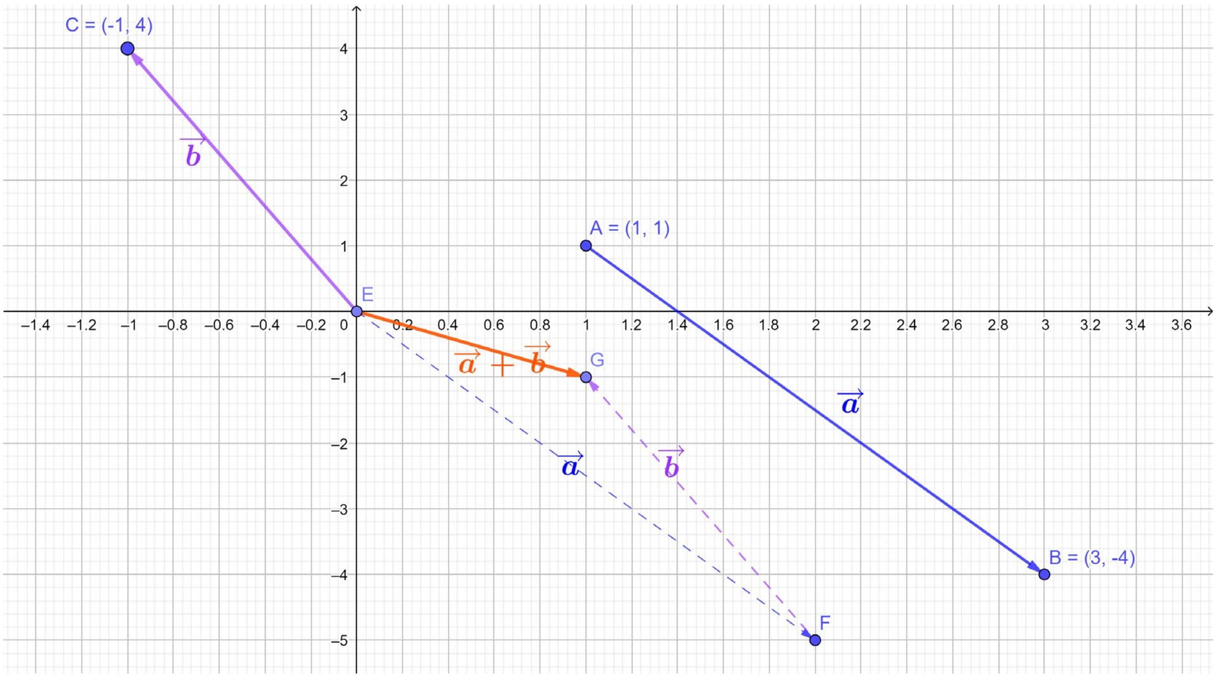

Figure 4.20 The component form of the vector $\vec{a}$ is $\vec{a} = (2, -5)$. In component form, $\vec{a} + \vec{b} = (1, -1)$

Dot or scalar product of vectors $\vec{a}$ and $\vec{b}$ is equal to:

$$\vec{a} \circ \vec{b} = a_x b_x + a_y b_y + a_z b_z .$$

The result from scalar product of two vectors is always a real number.

If the angle between two verctors $\vec{a}$ and $\vec{b}$ is 90°, then $\vec{a} \cdot \vec{b} = 0$, because $cos(90°) = 0$.

Angle between vectors $\vec{a}$ and $\vec{b}$ is calculated according to the formula for dot product:

$$cos \sphericalangle (\vec{a}, \vec{b}) = \frac{\vec{a} \circ \vec{b}}{|\vec{a}||\vec{b}|} = \frac{a_x b_x + a_y b_y + a_z b_z}{\sqrt{a_x^2 + a_y^2 + a_z^2} \cdot \sqrt{b_x^2 + b_y^2 + b_z^2}} .$$

Cross or vector product of vectors $\vec{a}$ and $\vec{b}$ can be calculated according to the formula

$$\vec{a} \times \vec{b} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ a_x & a_y & a_z \\ b_x & b_y & b_z \end{vmatrix} .$$

The result from cross or vector product of two vectors is always a vector.

Mixed triple product is calculated according to the formula:

$$\left(\vec{a}\times\vec{b}\right)\circ\vec{c}=\begin{vmatrix} a_x & a_y & a_z \\ b_x & b_y & b_z \\ c_x & c_y & c_z \end{vmatrix}$$

Example:

Let are given the vectors: $\vec{a}$ = (1, 0, 1) and $\vec{b}$ = (2, –2, 2). Determine the angle $\varphi = \sphericalangle(\vec{a}, \vec{b})$.

Solution:

$$cos\varphi=\frac{(1,0,1)\cdot(2,-2,2)}{\sqrt{1^2+0^2+1^2}\cdot\sqrt{2^2+(-2)^2+2^2}}=\frac{2+0+2}{\sqrt{2}\cdot\sqrt{12}}=\frac{4}{2\sqrt{6}}$$

$$\varphi=35°15'52''$$

Example:

Examine whether the vectors $\vec{a}$ = (2, –3,1) and $\vec{b}$ = (3, 1, 0) are perpendicular to each other.

Solution:

Two vectors are perpendicular to each other if and only if scalar product of these vectors is zero. Therefore,

$$\vec{a}\circ\vec{b}=6-3+0=3$$

<u>Answer:</u> The vectors are perpendicular to each other.

Example:

Determine area of a triangle that is spanned by vectors $\vec{a}$ = (–3, 2, –2) i $\vec{b}$ = (1, –4, 1).

Solution:

$$P_\Delta=\frac{1}{2}\left|\vec{a}\times\vec{b}\right|$$

$$\vec{a}\times\vec{b}=\begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ -3 & 2 & -2 \\ 1 & -4 & 1 \end{vmatrix}=-6\vec{i}+\vec{j}+10\vec{k}$$

$$P_{\Delta} = \frac{1}{2}\left|\vec{a} \times \vec{b}\right| = \frac{1}{2}\left|-6\vec{i} + \vec{j} + 10\vec{k}\right| = \frac{1}{2}\sqrt{36+1+100} = \frac{1}{2}\sqrt{137}$$

Example:

Given are points $A\ (1,2,1);\ B\ (3,-2,1);\ C\ (1,4,3)\ i\ D\ (5,0,5)$. Determine volumen V of the parallelopiped determined by vectors $\overrightarrow{AB}, \overrightarrow{AC}\ i\ \overrightarrow{AD}$.

$Solution$:

$\vec{a} = \overrightarrow{AB} = (2,-4,0)$

$\vec{b} = \overrightarrow{AC} = (0,2,2)$

$\vec{c} = \overrightarrow{AD} = (4,-2,4)$

$$(\vec{a} \times \vec{b}) \circ \vec{c} = \begin{vmatrix} 2 & -4 & 0 \\ 0 & 2 & 2 \\ 4 & -2 & 4 \end{vmatrix} = 8$$

$$V = |(\vec{a} \times \vec{b}) \circ \vec{c}| = |8| = 8$$