4.7 Connectedness and application in the maritime field

Example 1: A problem involving the bearing (direction) of a boat. A boat is traveling at a speed of 25 mph. The vector that represents the velocity is (√2, -√2). Determine the bearing of the boat.

SOLUTION:

Let's start by sketching the situation described (Figure 3.8).

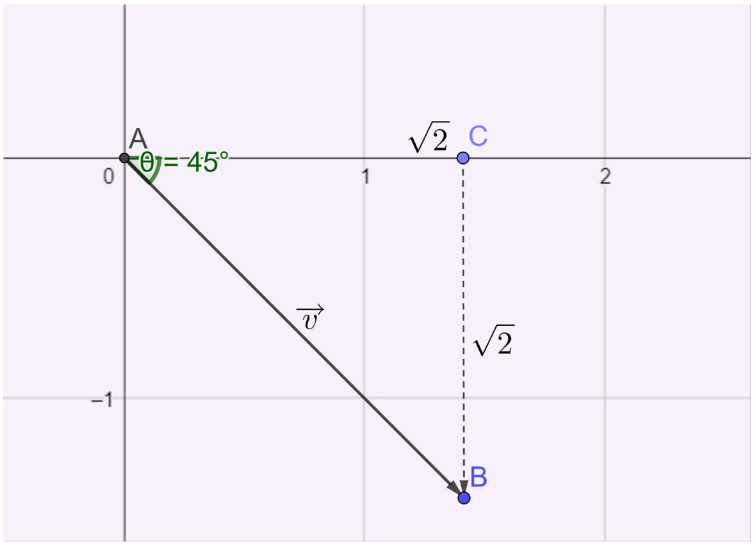


Figure 4.21 The component form of the velocity vector $\vec{v} = (\sqrt{2}, -\sqrt{2})$

Initially, the boat travels in the direction of velocity vector $v = (\sqrt{2}, \sqrt{2}) = \sqrt{2}\,\vec{\imath} - \sqrt{2}\,\vec{\jmath}$. Make a triangle ΔABC. It is the right triangle and the length of its cathetus $\overleftarrow{AC}$ and $\overleftarrow{AB}$ is $\sqrt{2}$.

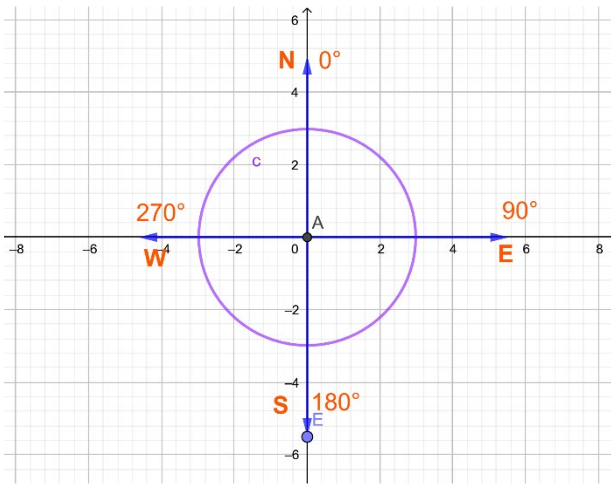
The **angle** θ is the bearing of the boat and it can be solved using tangent. Tangent is opposite cathetus over adjacent so we will get $\sqrt{2}$ over $\sqrt{2}$:

$$tg\theta = \frac{\sqrt{2}}{\sqrt{2}} = 1$$

$$\theta = arctg(1) = 45°$$

In navigationm, the bearing is described in the following way.

- Traveling due **north** it would be said the boat is at a bearing of **zero** degrees.
- If the boat travels due **east** it is at a bearing of **90** degrees.
- If the boat travels due **south** it is at a bearing of **180** degrees.



- If the boat travels due **west** it is at a bearing of **270** degrees.

So if the boat travels due east it would be at baring of 90 degrees. But actually, it travels at an extra $45°$ so $90° + 45° = 135°$. **The bearing $\boldsymbol{\theta}$ of the boat is 135°.**

Example 2: (https://www.geogebra.org/m/kmsTyU2S)

A ship leaves port on a bearing of 28° and travels 7.5 miles. The ship then turns due east and travels 4.1 miles. How far is the ship from the port and what is its bearing?

SOLUTION:

The following figure represents the described situation.

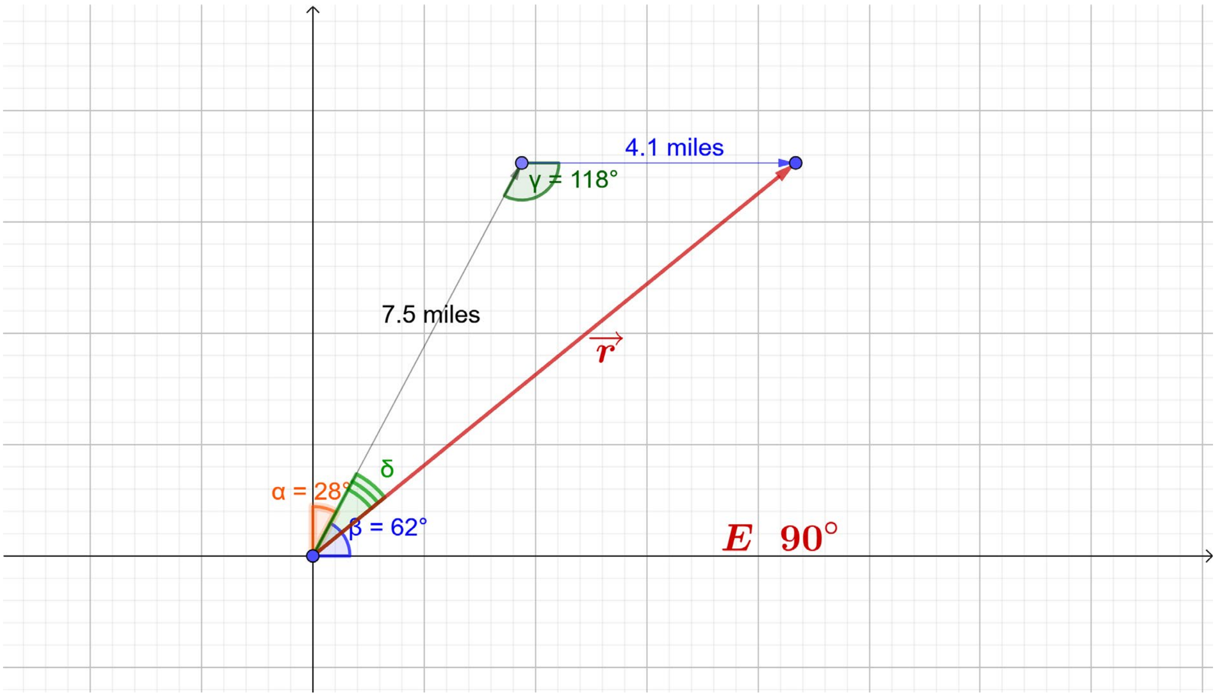


Figure 4.22 The component form of the resultant vector $\vec{r}$

The angle α is the bearing of the ship because the bearing is measured due to the north. In the direction α = 28°, the ship travels 7.5 miles and then it turns due east and travels in this direction for 4.1 miles. The distance between the current ship's position and the port would be determined as a magnitude of a resultant vector $\vec{\boldsymbol{r}}$.

To solve the problem it is needed to determine some more details.

If α is 28°, it implies $\beta = 90° - \alpha = 62°$.

The angle $\boldsymbol{\gamma}$ would have to be $\gamma = 180° - 62° = 118°$

The magnitude of the vector would be determined by the cosine law:

$$|\vec{r}|^2 = (7.5)^2 + (4.1)^2 - 2 \cdot (7.5)(4.1)cos(118°)$$

$$|\vec{r}|^2 = (56.25) + (16.81) - 61.5 \cdot (-0.469)$$

$$|\vec{r}|^2 = 101.93$$

$$|\vec{r}| = \sqrt{101.93} = 10.09$$

The ship is far 10.1 miles from the port.

Ship's bearing would be determined as sum of the angles $\boldsymbol{\alpha}$ and $\boldsymbol{\delta}$.

The angle $\boldsymbol{\delta}$ would be calculated from the sinus law as follows:

$$\frac{sin(118°)}{10.1} = \frac{\boldsymbol{sin(\delta)}}{4.1}$$

$$\frac{0{,}8829}{10.1} = \frac{\boldsymbol{sin(\delta)}}{4.1}$$

$$0.0874 = \frac{\boldsymbol{sin(\delta)}}{4.1} \Rightarrow \boldsymbol{sin(\delta)} = 0.0874 \cdot 4.1 = 0.3584$$

$$\boldsymbol{\delta = arcsin(0.3584) = 21°}$$

The ship's bearing is $\boldsymbol{\alpha + \delta = 28° + 21° = 49°}$.

Example 3:

A sailboat under auxiliary power is sailing on a bearing 25° north of west at 6.25 mph. The wind is blowing in the direction 35° south of west at 15 mph.

SOLUTION:

This navigation problem uses variables like speed and direction to form vectors for computation. Like with an aircraft, some navigation problems ask us to find the ground speed of a boat using the combined the force of the wind which affects on the boat and the speed of the boat. For these problems it is important to understand the resultant of two forces and the components of force.

In our case, the sailboat is sailing in the direction 25° north of west (NW) or in direction of the speed vector $\vec{s}$ (sailboat vector). It is sailing at 6.26 mph which would have to be interpreted as the magnitude of the vector $\vec{s}$. There is a 15 mph wind's current blowing in the direction 35° south of west (SW). On the figure below this force is represented as the vector $\vec{w}$. If there is no wind the sailboat will be sailing in the direction of its course or 15 degrees.

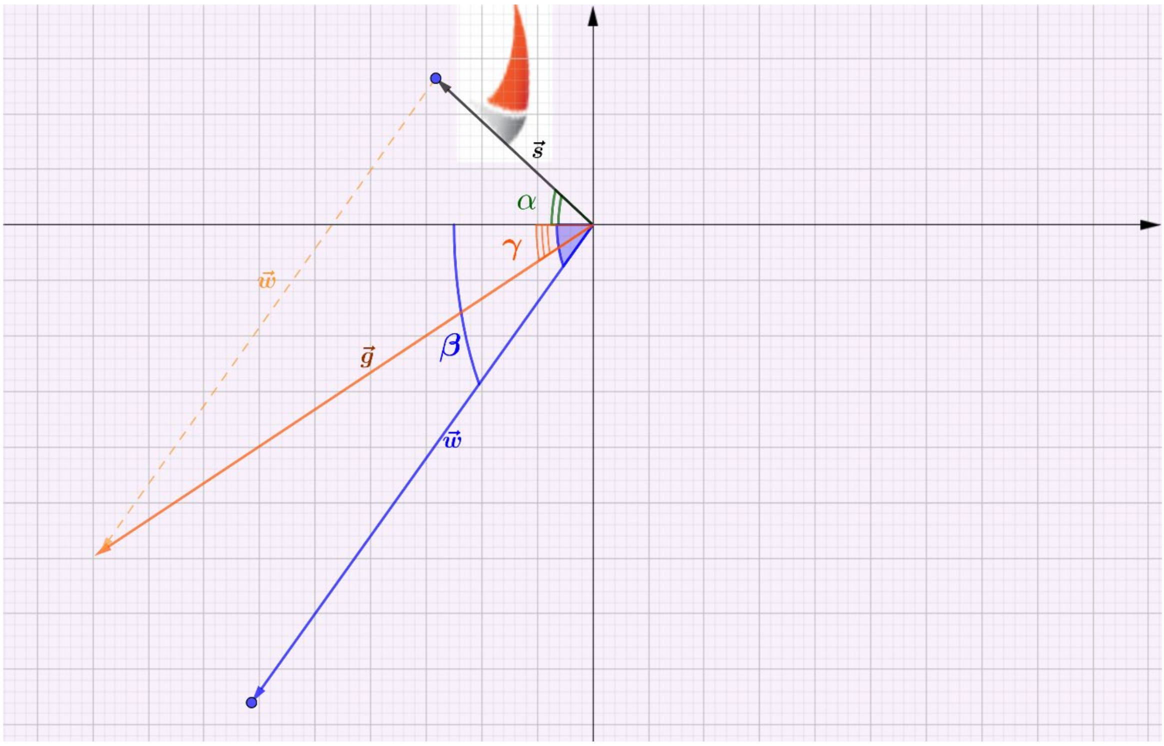


Figure 4.23 The vector of actual (ground) sailboat's velocity is denoted by $\vec{g}$ and it is the resultant of the sailboat vector plus the wind vector.

But the wind is blowing in the direction of the vector $\vec{w}$ and if you do not compensate sailboat's course, the sailboat will be pushed by the wind and its actual course would be to the down or SW of destination. Probably, it will affect how fast the sailboat is sailing.

Vectors will help us to determine exactly where the sailboat is sailing and how fast. It would be done through the vector addition.

If we consider adding the sailboat vector $\vec{s}$ to the wind vector $\vec{w}$, geometrically it means that it is possible to move wind vector $\vec{w}$ in the sense that these two vectors have the same start point. On the figure, this moved vector is noted by the orange colour and it is also wind vector. The resultant of these vectors (the red vector on the figure) is the sailboat vector plus the wind vector. This resultant vector is something what we are interested in right because this red vector is real direction where the sailboat is sailing. It is called the actual or ground course and it is noted by $\vec{g}$.

It is needed to find the speed of sailboat as the magnitude of the vector $\vec{g}$ and the direction of the sailboat by finding the angle γ.

The first part of problem is to determine the component of the sailboat vector $\vec{s}$ and the wind vector $\vec{w}$.

It can be done on the following way:

$$\vec{s} = \left(6.25 \cdot cos(180° - 25°), 6.25 \cdot sin(180° - 25°)\right).$$

Note that we must take the angle from second quadrant because the sailboat is sailing north of west. It is the reason why the angle of sailboat vector is determined as $180° - 25°$.

Calculating it is possible to find the components

$$\vec{s} = (-5.66, 2.64)$$

The wind vector lies in the third quadrant so its angle will be $180° + 35° \ = \ 215°$.

The components of the wind vector can be calculated as follows:

$$\vec{w} = \left(15 \cdot cos(215°), 15 \cdot sin(215°)\right) = (-12.29, -8.60)$$

$$\vec{g} = \vec{s} + \vec{w} = (-5.66, 2.64) \ + \ (-12.29. -8.60) = \left(-5.66 + (-12.29), 2.64 + (-8.60)\right)$$

$$\vec{g} = (-17.95, -5.96)$$

The speed of the sailboat is the same as the magnitude of the vector $\vec{g}$.

$$||\vec{g}|| = \sqrt{(-17.95)^2 + (-5.96)^2}$$

$||\vec{g}|| = \sqrt{322{,}2025 + 35{,}5216} \ = \sqrt{357{,}724} = 18.91359 \approx 18.91 =$

The actual (ground) speed of the sailboat is around 19.91 mph.

The second part of our problem is to determine the actual course (course over ground).

$$tg(\gamma) = \frac{-5.96}{-17.95} = 0.3320334 \ \ (III \ quadrant)$$

$\gamma = arctg(0.3320334) = 18.37° + 180° \ (III \ quadrant)$

We can say that the sailboat is sailing in the direction 18.37° south of the west.

Example 4:

A large ship has gone aground in the harbour and two tugboats, with cables attached, attempt to pull it free. If one tug pulls in the direction 38° north of east with a force of 2300 lbs and the second tug plus in the direction 9° south of the east with the force of 1900 lbs. Find the direction and magnitude of the resultant force.

SOLUTION:

The problem is sketched in the figure 3.11.

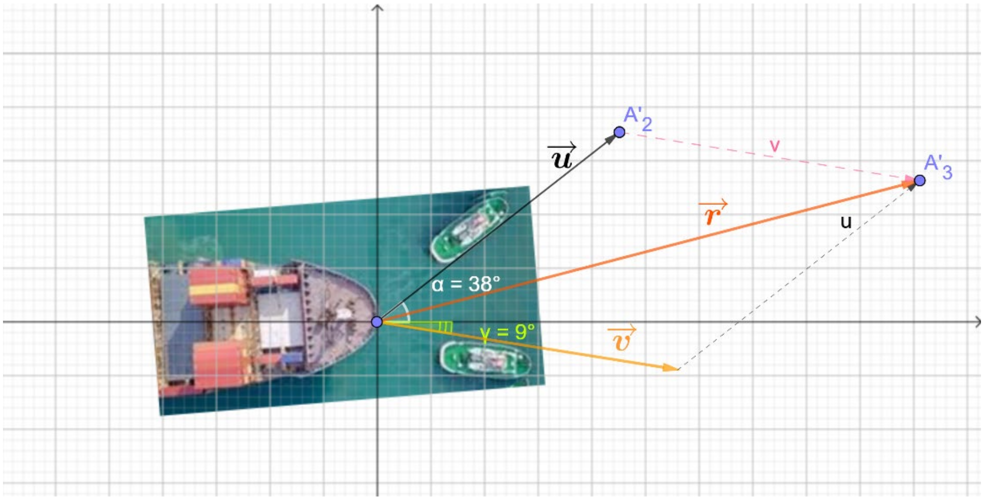


Figure 4.24 The tugboat vectors

The first step is to determine the components of tugboat vectors $\vec{u}$ and $\vec{v}$.

$$\vec{u} = \left(2300 \cdot cos(38°), 2300 \cdot sin(38°)\right) = (1812.42, 1416.02\ )$$

$$\vec{v} = \left(1900 \cdot cos(-9°), 1900 \cdot sin(-9°)\right) = (1876.60, -297.23\ )$$

$$\vec{r}\ = \vec{u} + \vec{v} = (3689.028, 1118.79\ \ )$$

The next step is to determine the magnitude and the direction of vector $\vec{r}$.

Magnitude: $\overrightarrow{\|r\|}\ = \sqrt{3689.028^2\ +\ 1118.79^2}\ = \sqrt{14.860.551{,}52}$= 3854.938

Direction: $arctg\left(\frac{1118.79}{3689.028}\right) = arctg(0{,}3032750) = 16.87°\ north\ of\ east$

Example 5:

Navigational plotting

Ship A sails on course $K\ = 056°$, at the speed of 14.5 knots. In time $t\ = 13{:}16$ the officer of the watch observes on the radar display the reflection of another vessel B. The OOW proceeds with observing the ship B and obtains the following data:

tx = 13:20 taken the bearing to ship B $\boldsymbol{\omega 1}\ = \mathbf{093°}$ at a distance $\boldsymbol{D1}\ = \mathbf{8.5}\ \boldsymbol{NM}$,

tx = 13:26 taken the bearing to ship B $\boldsymbol{\omega 2}\ = \mathbf{093°}$ at a distance $\boldsymbol{D2}\ = \mathbf{8}\ \boldsymbol{NM}$.

The data recorded are represented in the picture below ($\boldsymbol{M}\ \mathbf{1}\ \boldsymbol{cm}\ = \mathbf{1}\boldsymbol{NM}\ = \mathbf{1}\ \boldsymbol{kt}$) :

Solution obtained by using true plotting:

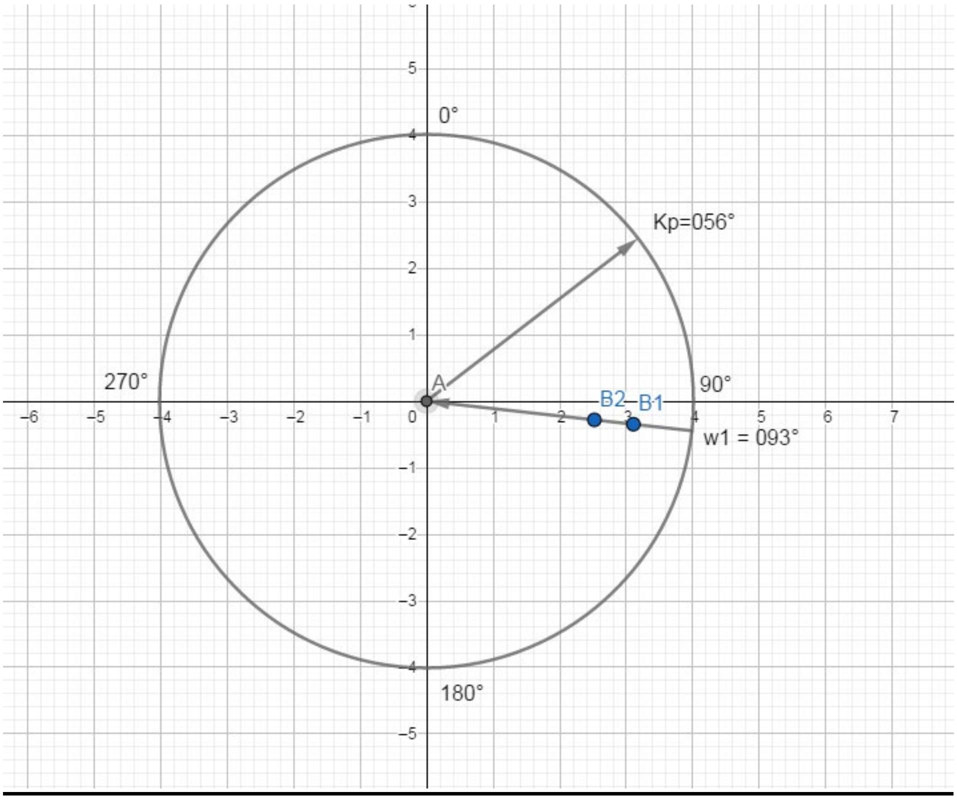


Figure 4.25

Ship (B) sails on course $K = 318°$, at the speed of $v = 14.2\,kt$
- ASPECT – the angle between the bearing line and the other ship's course line, $A = 44° = const.$, $\omega = 093° = const.$

Conclusion – since the recording bearing of ship B does not change in the time lapse of 6 minutes, and the distance between the ships A and B decreases, the ships are on collision courses, and if the avoiding manoeuvre prescribed in the Collision Regulations is not undertaken, the ships will collide.

Example 6:

Effect of ocean current on navigation

The figure shows the easiest way to solve the problem of navigation with the current. A ship sails from position $P1$ on course $Kp = 110º$ towards the port of Zadar. After having travelled 3 NM the ship was supposed to be in the DR position $Pz1$, but the measurements found that, due to drift, the ship was not in that position, but in position $P2$. The magnitude of the drift due to the ocean current can be calculated if the positions $Pz1$ and $P2$ are connected by vector showing by how much the ship went off course after travelling 3 NM. Although the course held on the compass was 110 º (course through the water), it can be seen that the ship was sailing in the true course 103 º (course over ground). If from position P_2 the course is plotted towards Zadar, it will be 117 º (planned course over ground), and if the ocean current had no effect, after travelling the next 3 NM the ship would be in position $Pz2$. However, since the current has an effect, the ship will again go off course and will find itself in position differing from $Pz2$ by the magnitude of the drift vector (drift vector is marked red in colour). Therefore, from $Pz2$ using the triangle and dividers, the drift vector is transferred, but in the opposite direction, and the course towards Zadar is obtained if the position P_2 is connected with the peak of the plotted drift vector. Thus, the course 123º is obtained (planned course through the water), and if the ship proceeds on the course planned, it will reach the planned point of arrival.

Direction and strength of the current can be detrmined so that with the use of nautical triangles the direction of the current is determined in degrees, and its strength can be detrmined so that the current vector is taken in the dividers and from the chart the strength in NM is read against the latitude scale.