COMPLEX NUMBERS

1. The imaginary unit *i*

The imaginary unit is defined as

$$i = \sqrt{-1}, \quad \text{where } i^2 = -1.$$

2. Complex Numbers and Imaginary Numbers

The set of all numbers in the form $a + bi$ with real numbers a and b and i the imaginary unit, is called the set of **complex numbers**.

The real number a is called the **real part** and the real number b is called the **imaginary part** of the complex number $a + bi$.

If $b \neq 0$ then the complex number is called an ***imaginary number***. An imaginary number in the form bi is called a ***pure imaginary number***.

3. Operations with Complex Numbers

1) $(a + bi) + (c + di) = a + bi + c + di = (a + c) + (b + d)i$
2) $(a + bi) - (c + di) = a + bi - c - di = (a - c) + (b - d)i$
3) $(a + bi)(c + di) = ac + adi + bci + bdi^2 = ac + adi + bci - bd = (ac - bd) + (ad + bc)i$

EXAMPLE 1:

1) $(2 + 5i) + (3 - 9i) = 2 + 5i + 3 - 9i = (2 + 3i) + (5i - 9i) = 5 - 4i$
2) $(1 + 3i) - (7 - 2i) = 1 + 3i - 7 + 2i = (1 - 7) + (3i + 2i) = -6 + 5i$
3) $(1 - 2i)(3 + 4i) = 3 + 4i - 6i - 8i^2 = 3 + 4i - 6i + 8 = 11 - 2i$

4. Complex Conjugates and Division

The complex conjugate of the number *a* + *bi* is

a - *bi*

and the complex conjugate of *a* – *bi* is

a + *bi*.

The multiplication of complex conjugates gives a real number.

$$(a + bi)(a - bi) = a^2 + b^2$$

$$(a - bi)(a + bi) = a^2 + b^2$$

Complex conjugates are used to divide complex numbers. The goal of the division procedure is to obtain a real number in the denominator.

This real number becomes the denominator of *a* and *b* in the quotient *a* + *bi*.

By multiplying the numerator and the denominator of the division by the complex conjugate of the denominator, you will obtain this real number in the denominator.

$$\frac{a+bi}{c+di} = \frac{(a+bi)(c-di)}{(c+di)(c-di)} = \frac{ac-adi+bci-bdi^2}{c^2-d^2i^2} = \frac{(ac+bd)+(bc-ad)i}{c^2+d^2}$$

$$= \frac{ac+bd}{c^2+d^2} + \frac{bc-ad}{c^2+d^2}i$$

$c^2 + d^2 \neq 0$

EXAMPLE 2:

1) $\frac{2-5i}{3+i} = \frac{(2-5i)(3-i)}{(3+i)(3-i)} = \frac{6-2i-15i+5i^2}{9-i^2} = \frac{6-2i-15i-5}{9+1} = \frac{1-17i}{10}$

2) $\frac{1+4i}{5-2i} = \frac{(1+4i)(5+2i)}{(5-2i)(5+2i)} = \frac{5+2i+20i+8i^2}{25-4i^2} = \frac{5+2i+20i-8}{25+4} = \frac{-3+22i}{29}$