Integration by parts

$$\int u\,dv = u \cdot v - \int v\,du$$

1) For given integrals **choose** the functions for ***u*-substitution**:

$x;\ \ 8x;\ \ 2x+1;\ \ x^2;\ \ sinx;\ \ cosx;\ \ cos^2 6x;\ \ e^x;\ \ lnx;\ \ lgx;\ \ arcsinx;\ \ arctanx$

2) **Solve** following integrals **1; 3; 4; 6**

Nr	Integral	*u*-function
1	$\int x^2 cosx\,dx$	$u = x^2$
2	$\int 8x\ sinx\,dx$	
3	$\int x\ arctanx\,dx$	
4	$\int (2x+1)e^x dx$	
5	$\int 8x\ lnx\,dx$	
6	$\int \frac{lgx}{x^2}\,dx$	
7	$\int e^x sinx\,dx$	
8	$\int \frac{6x}{cos^2 6x} dx$	
9	$\int (2x+1) lnx\,dx$	
10	$\int arcsinx\,dx$	
11	$\int x^2 lnx\,dx$	

Answers

1)

Nr	Integral	u-function
1	$\int x^2 cosx\ dx$	$u = x^2$
2	$\int 8x\ sinx\ dx$	$u = 8x$
3	$\int x\ arctanx\ dx$	$u = arctanx$
4	$\int (2x+1)e^x dx$	$u = 2x + 1$
5	$\int 8x\ lnx\ dx$	$u = lnx$
6	$\int \frac{lgx}{x^2}\ dx$	$u = lgx$
7	$\int e^x sinx\ dx$	$u = sinx\ or\ u = e^x$
8	$\int \frac{6x}{cos^2 6x} dx$	$u = x$
9	$\int (2x+1)lnx\ dx$	$u = lnx$
10	$\int arcsinx\ dx$	$u = arcsinx$
11	$\int x^2 lgx\ dx$	$u = lgx$

2) Solutions

1. $\int x^2 cosx\ dx = \begin{vmatrix} u = x^2 & du = 2xdx \\ dv = cosxdx & v = \int cosx\ dx = sinx \end{vmatrix} =$

$= x^2 sinx - \int 2xsinx\ dx = \begin{vmatrix} u = 2x & du = 2dx \\ dv = sinxdx & v = \int sinxdx = -cosx \end{vmatrix} =$

$$= x^2 sinx - \left(-2xcosx + 2\int cosxdx\right) =$$

$$= x^2 sinx + 2x\, cosx - 2sinx + C$$

3. $$\int x\, arctanx\, dx = \begin{vmatrix} u = arctanx & du = \dfrac{dx}{1+x^2} \\ dv = xdx & v = \int xdx = \dfrac{x^2}{2} \end{vmatrix} =$$

$$= \frac{x^2}{2} arctanx - \frac{1}{2}\int \frac{x^2}{1+x^2} dx =$$

$$= \frac{x^2}{2} arctanx - \frac{1}{2}\int \frac{x^2 + 1 - 1}{1+x^2} dx =$$

$$= \frac{x^2}{2} arctanx - \frac{1}{2}\int dx + \frac{1}{2}\int \frac{dx}{1+x^2} =$$

$$= \frac{x^2}{2} arctanx - \frac{1}{2}x + \frac{1}{2} arctanx + C$$

4. $$\int (2x+1)e^x dx = \begin{vmatrix} u = 2x+1 & du = 2dx \\ dv = e^x dx & v = \int e^x dx = e^x \end{vmatrix} =$$

$$= (2x+1)e^x - 2\int e^x dx =$$

$$= (2x+1)e^x - 2e^x + C$$

6. $$\int \frac{lgx}{x^2}\, dx = \begin{vmatrix} u = lgx & du = \dfrac{1}{xln10} dx \\ dv = \dfrac{dx}{x^2} & v = \int \dfrac{dx}{x^2} = \dfrac{-1}{x} \end{vmatrix} =$$

$$= -\frac{1}{x} lgx + \frac{1}{ln10}\int \frac{1}{x}\cdot\frac{1}{x} dx =$$

$$= -\frac{1}{x} lgx - \frac{1}{xln10} + C$$