5.1. FUNCTION. BASICS. GRAPH. FUNCTION TABLE

The function is one of the basic concepts in mathematics. In this section, a formal definition of a function is provided. Functions can be represented through tables, formulas, and graphs The formal notation and terms related to functions are provided.

The notion of notation of functions becomes clear with the help of examples. The section contains several maritime examples of using functions, using GeoGebra and MS Excel.

Definition (function):

Function f is a relation in which each object from the set of inputs X is associated to exactly one object from the set of outputs Y. Each function must have three elements defined:

1. Domain X – a set of inputs, i.e. a set of all arguments of the function
2. Mapping rule f – the way this data is transformed - functional equation
3. Codomain Y – a set of possible outputs

Element x is an ***argument*** (input) and element y is an ***image*** of x (the value of function or output).

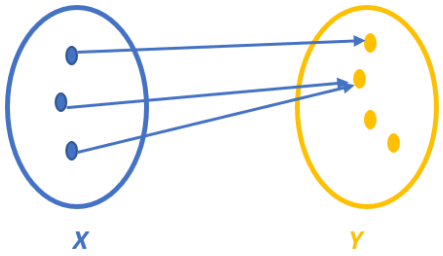


Figure 5.1

The statement can be written

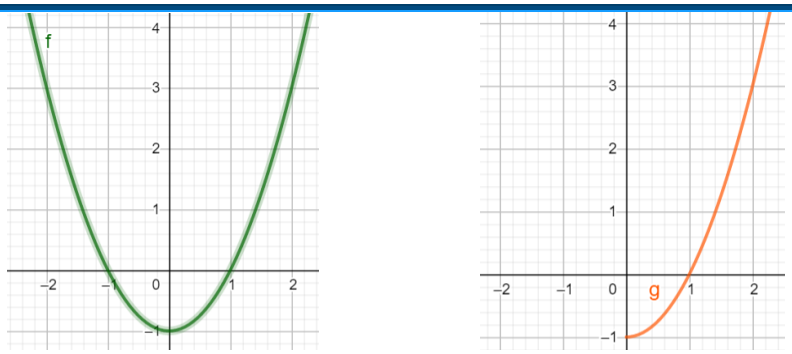
$$f: X \to Y$$

Set X is called the domain, set Y is called the codomain.

Functions where domain and codomain are subsets of real number sets are commonly used in real life problems.

Functions $f: \mathbb{R} \to \mathbb{R}$ defined by $f(x) = x^2 - 1$ and $g: \langle 0, \infty \rangle \to \langle -1, \infty \rangle$ defined by $g(x) = x^2 - 1$ are not the same functions, as we can see on Figure 5.2

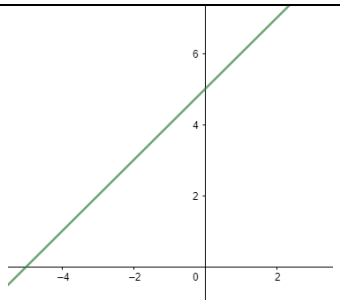
Co-funded by the
Erasmus+ Programme
of the European Union



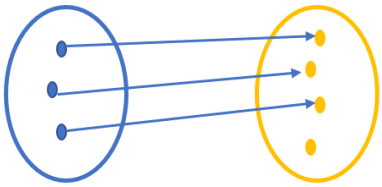
a) b)

Figure 5.2. Graph of functions: a) $f(x) = x^2 - 1$; *b)* $g(x) = x^2 - 1$

There are various ways to assign functions. The table below shows three different ways to set the function $f(x) = x + 5$.

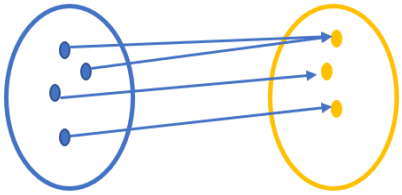
Descriptive	Table	Graph
Add the number 5 to each real number.	X | y 1 | 6 2 | 7 3 | 8 4 | 9	

Injection is a one-to-one function. It is a function that maps distinct elements of the domain to distinct elements of the codomain.

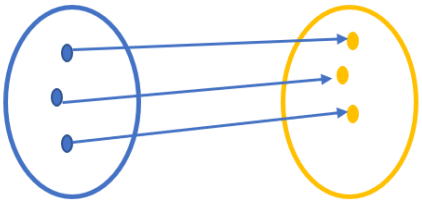


A function is called *surjection* if every element of the codomain is mapped by at least one element of the domain.

Co-funded by the Erasmus+ Programme of the European Union



A function is called ***bijection*** if a function is both injection and surjection.



Example 5.1

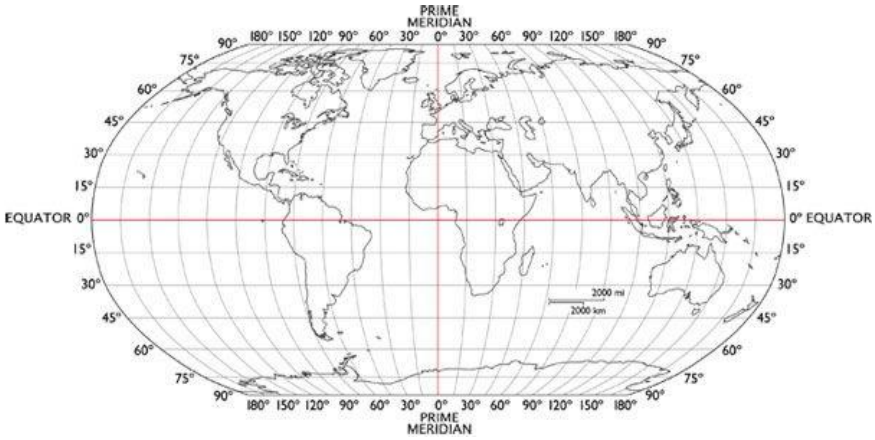


Figure 5.3

Global positioning using longitude and latitude is an example of a function. Each point on the Earth's surface is associated with a unique combination of two coordinates.

For example, the city of Split is associated with coordinates (43.5081°N, 16.4402°E) (43° 30’ 29’’ N and 16° 26’ 25’’ E).

On Google maps https://www.google.com/maps choose the place you want to travel to next summer and read its longitude and latitude. Convert the values to degrees, minutes and seconds.

Is the mapping that accompanies each point on the Earth's surface an ordered pair of number coordinates:

a) function

b) injection

Co-funded by the
Erasmus+ Programme
of the European Union

c) surjection

d) bijection.

Solution:

Computer instructions: right click on a place on the map, select *What's here*? Latitude and longitude in degrees will be displayed.

Mobile phone instructions: Press and hold the selected place on the screen. The latitude and longitude in degrees will be displayed in the data entry bar.

You can check the calculation on your computer by clicking on the amount of latitude or longitude. Latitude and longitude in degrees, minutes, and seconds will be displayed in the left corner of the data entry bar.

The mapping that accompanies each point on the Earth's surface an ordered pair of number coordinates is a function, injection, surjection and bijection.

a) The mapping is a function, because each point on the surface has corresponding coordinates.

b) The function is an injection because two different places on the Earth's surface will surely have different coordinates. There cannot be two different cities with the same latitude and longitude.

c) The function is a surjection because all possible values of latitude and longitude are hit.

If we would somehow add another meridian and have 181° of longitude, that would be a problem. Then no place on Earth would be mapped to that 181° longitude, and the function would no longer be a surjection.

d) Since the function is an injection and surjection, it is a bijection.

This is important because it means we can go "backwards". Knowing the coordinates of one place, for instance (43.5081°N, 16.4402°E), we can distinctively say which point on the surface it is (Split).

Example 5.2

In a port, 300 passengers can board one of the 3 ships (A, B or C) that depart at the same time. Each ship has at least 2 passengers.

Is the mapping in which each passenger is associated to a ship:

a) function

b) injection

c) surjection

d) bijection

e) Is the reverse mapping assigned to each ship a list of passengers on it a function?

Solution:

Co-funded by the
Erasmus+ Programme
of the European Union

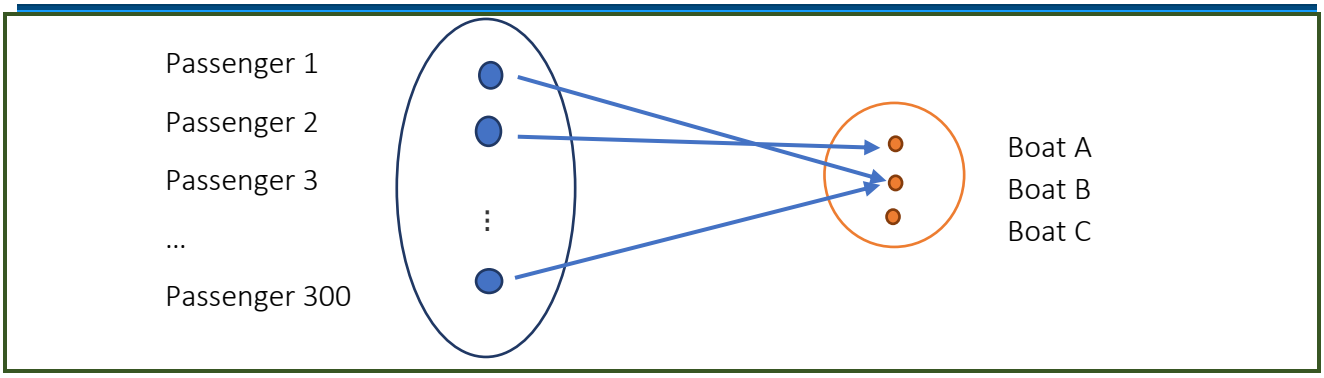


Figure 5.4

a) The mapping is a function because every passenger will board on exactly one ship.

b) The mapping is not an injection, because it is not one-to-one mapping. There are 300 hundred passengers and only 3 ships, so several passengers' board on the same ship.

c) The mapping is a surjection, because every element of the codomain (every ship) has at least one associated element of the domain (passenger).

d) The mapping is not a bijection, because it is not an injection.

e) Reverse mapping is not a function because every ship is not associated to exactly one passenger, but to many passengers.

Example 5.3

The GPS system receives messages about the coordinates of the Nautilus ship every hour during the four-hour voyage.

The last voyage of the ship is given by the table

Time	X coordinate (Northern latitude)	Y coordinate (Eastern longitude)
12:00	44.52	14.51
13:00	44.52	14.62
14:00	44.52	14.69
15:00	44.52	14.81
16:00	44.52	14.89

The ship's voyage was sketched in Geogebra. https://www.geogebra.org/m/shtqu5kq In which direction did the ship move? What was its course?

Co-funded by the Erasmus+ Programme of the European Union

If we know that the total length of the voyage was 24 km, what was the average speed of the ship?
Did the ship have a steady speed during the voyage?
Then did the ship go the fastest and when the slowest?
Determine the speed of the ship in each of the intervals.
Try to determine the Y coordinate of the ship in each time interval when the ship would be moving at a constant velocity of 6 km/h.
g) If 1 knot = 1.85 km/h, what was the average velocity of the ship in knots

Solution:

a) from Rovenska Nova to Novalja

b) 6 km/h

c) No

d) Ship had the fastest interval between 14:00 and 15:00, and slowest between 13:00 and 14:00.

Time	ΔY coordinate (longitude)	ΔY coordinate (km)	v (km/h)
12:00 – 13:00	0.11	6.95	6.95
13:00 – 14:00	0.07	4.42	4.42
14:00 – 15:00	0.12	7.58	7.58
15:00 – 16:00	0.08	5.05	5.05

f)

Time	Y coordinate (Eastern longitude) (real values)	Y coordinate (Eastern longitude) (with constant velocity 6/km/h)
12:00	14.51	14.51
13:00	14.62	14.605
14:00	14.69	14.70
15:00	14.81	14.795
16:00	14.89	14.89

g) $v = \frac{6}{1.85} = 3.24\ knots$

Co-funded by the Erasmus+ Programme of the European Union