

5.2. ANALYSING THE GRAPH OF A FUNCTION

For a given function $f(x)$, set of all pairs $(x, f(x))$ is called the graph of the function.

5.2.1. Sign of a function value

The positive regions of a function are intervals where the function is above the x-axis. Mathematically speaking, function is positive on interval $\langle a, b \rangle$ if $f(x) > 0$ for every $x \in \langle a, b \rangle$.

The negative regions of a function are intervals where the function is below the x-axis. Function is negative on interval $\langle a, b \rangle$ if $f(x) < 0$ for every $x \in \langle a, b \rangle$.

All points for which $f(x) = 0$ are called **zeros**.

Example 5.4 <https://www.geogebra.org/calculator/rvnautep>

Find the positive and negative regions of a function.

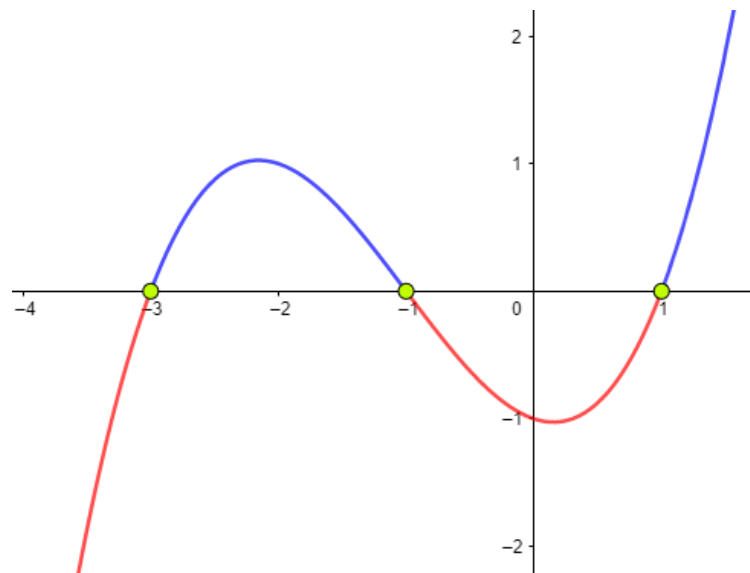


Figure 5.5

Solution:

Recall that we read function values on the y axis, so for a positive sign we are interested in x values where the y coordinate of that point is greater than 0.

All points colored blue have a positive functional value, so we say that the function is positive at these intervals. In this task these are intervals $\langle -3, -1 \rangle \cup \langle 1, +\infty \rangle$.

All points colored red have a negative functional value, so we say that the function is negative at these intervals. In this task these are the intervals $\langle -\infty, -3 \rangle \cup \langle -1, 1 \rangle$.

5.2.2. Increasing and decreasing functions

Function is increasing if when x increases, then y also increases. When $x_1 < x_2$ then $f(x_1) \leq f(x_2)$ we say that function is increasing.

Function is decreasing if when x increases, then y decreases. When $x_1 < x_2$ then $f(x_1) \geq f(x_2)$ we say that function is decreasing.

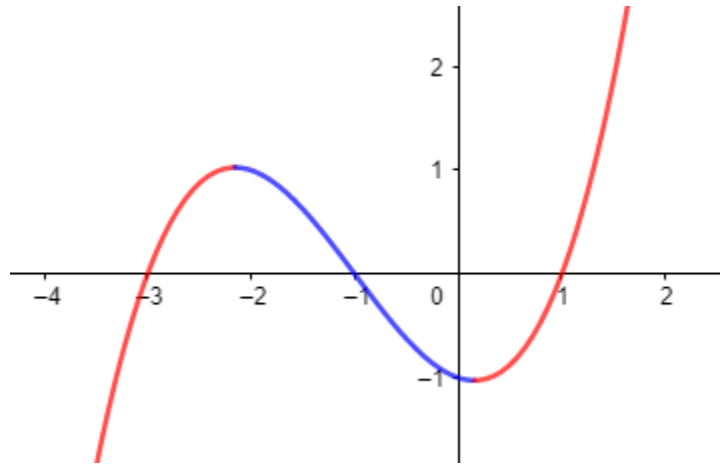


Figure 5.6 <https://www.geogebra.org/calculator/mufqrbvs>

For all values colored red the function is increasing.

For all values colored blue the function is decreasing.

Example 5.5

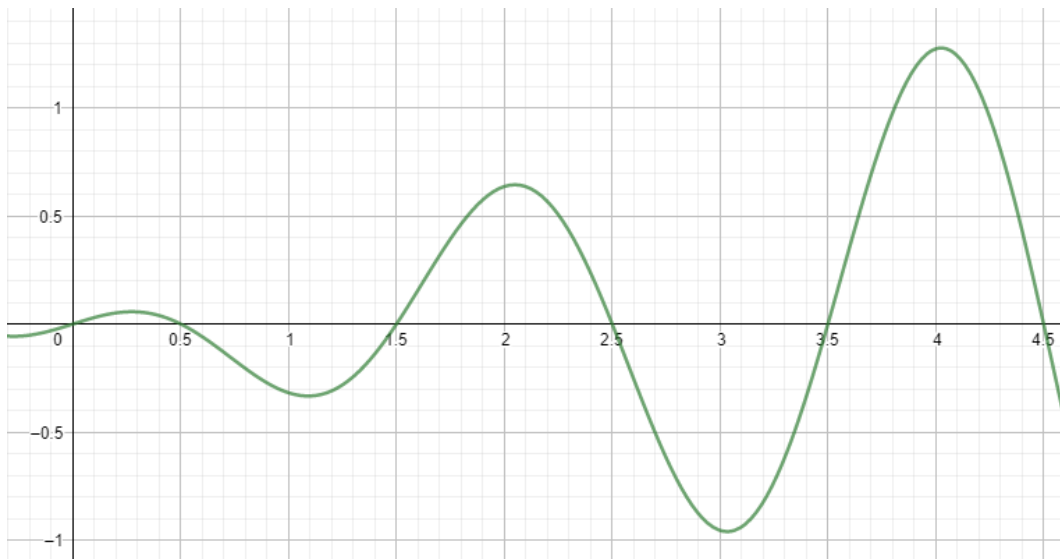


Figure 5.7

Find in interval $[0,4.5]$

All intervals where the function is positive (all x values for which the function value is positive)

All zero points

All intervals where the function is decreasing.

Solutions:

- a) $x \in \langle 0,0.5 \rangle \cup \langle 1.5,2.5 \rangle \cup \langle 3.5,4.5 \rangle$
- b) Zero points are $(0,0), (0.5,0), (1.5,0), (2.5,0), (3.5,0), (4.5,0)$
- c) $x \in \langle 0.5,1.5 \rangle \cup \langle 2.5,3.5 \rangle$

