## 5.3.  ODD AND EVEN FUNCTIONS

<span style="color:red">Aims</span>

1) Introducing concept of odd/even function

2) Students learn how to differentiate odd/ even function analytically and graphically.

3) Understand the concept of drawing function graphics.

4) Introducing algebraic and polar forms

### 5.3.1.  Even function

**Definition:**

A function is "**even**" when $f(-x) = f(x)$ for all $x$. When you plug in $-x$, you get back the same function with which you started.

*Example 5.6*

Are given function $f(x) = 3x^2 - 5x^4$ odd or even?

$f(-x) = 3(-x)^2 - 5(-x)^4 = 3x^2 - 5x^4$

$f(-x) = f(x)$ given function is even.

Even functions are symmetric about the y-axis. Graphics remain unchanged when reflected across the *y*-axis.

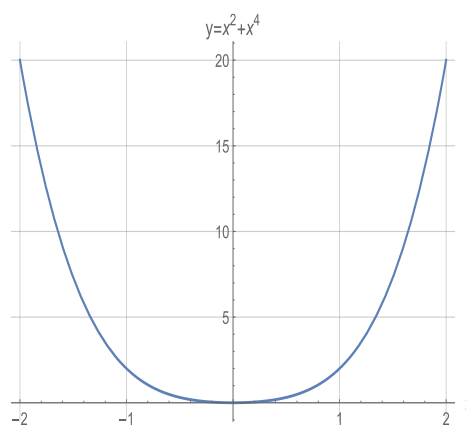**Graphic 1:** Even function $y = x^4 + x^2$



*Figure 5.8 Even function $y = x^4 + x^2$*

### 5.3.2.    Odd function

**Definition:**

A function is "odd" when f (- x) = - f (x) for all x. When you plug in -x, you get back the negation of the function with which you started.

*Example 5.7*

Are given function f(x)= $4x^3 + 7x$ odd or even?

f(-x) = $4(-x)^3 + 7(- x) = -4x^3 - 7x = - (4x^3 + 7x)$

Not even        Odd

Odd functions are symmetric about the origin. The graph remains unchanged after a rotation of 180 degrees about the origin.
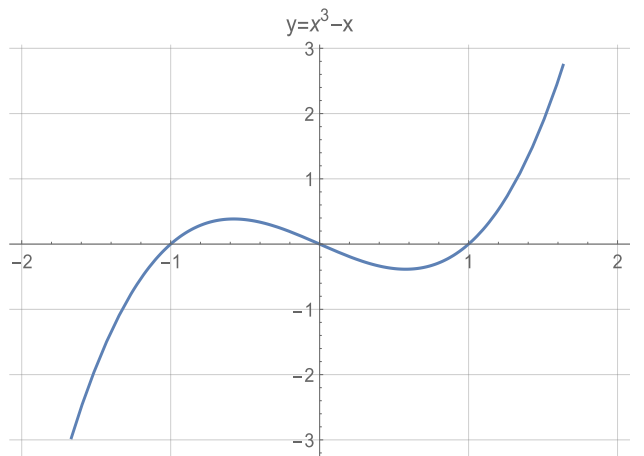
**Graphic 2:** Odd function y= $x^3$ - x



*Figure 5.9 Odd function y= $x^3$ - x*

1) *Neither odd nor even functions*

**Majority of functions** are neither odd nor even functions. All of these "other" functions are referred to as "neither", when being compared to the odd and even function definitions.

*Example 3:* Are given function  f(x) = $5x^2 + 3x + 5$ odd or even?

f( -x) = $5(-x)^2 + 3 (- x) + 5 = 5x^2 - 3x + 5 = - (5x^2 + 3x - 5)$

*f(-x) ≠ f(x)*  ⟹  $5x^2 + 3x + 5 ≠ 5x^2$ - 3x + 5  ⟹ Not even

f(-x)≠ -f(x)  ⟹ $(5x^2 + 3x - 5) ≠ - (5x^2 + 3x + 5)$  ⟹ Not odd

This function is neither.

### 5.3.3. Exercises

**Task 5.1** Is this function odd, even or neither

1) $y = x^2 + 5x;$     2) $y = -x^2 + 2;$     3) $y = x^2 + 2x - 3;$

4) $y = 5x - 2;$     5) $y = x^4 + 2x^2;$     6) $y = x^3 - 5x + 3;$

7) $y = x + \dfrac{1}{x};$     8) $y = \dfrac{2x^5}{3x^3 + x};$     9) $y = \dfrac{3}{x^2 - 5};$

10) $y = x^4 + x^2 - 3x;$     11) $y = x^3 + x^{-1};$     12) $y = 3x^4 + x^2 + 4.$

**Task 5.2** Is this function odd, even or neither

a) $y = 5x;$     e) $y = 5x^2;$     i) $y = 5x - 1;$     m) $y = \dfrac{3}{x};$

b) $y = 5x^2 + 1;$     f) $y = 6x;$     j) $y = 6x^2 + 4;$     n) $y = 6x + 1;$

c) $y = 2x^3;$     g) $y = 5x^2 + 1;$     k) $y = x^2 - 2x + 3;$     o) $y = 3x^4;$

d) $y = 7x^2 + x;$     h) $y = 3x^4 + x^2 + 4;$     l) $y = x^3 - 5x - 3;$     p) $y = x + \dfrac{1}{x}.$

### 5.3.4. Homework

1. $y = 2x^2 - 3x + 6$

2. $y = x^2 + 3x + 1$

3. $y = 3x^2 + 5x - 4$

4. $y = x^2 + x + 2$

5. $y = 3x^2 - 7x - 6$