Exercises

Task 5.18 Solve the equations:

a) $3^{x+2} - 3^{x-1} = \frac{26}{9}$.

b) $3 \cdot 5^x - 2 \cdot 5^{x-1} = 5^{x+1} - \frac{12}{5}$.

c) $\frac{3}{10} \cdot \left(\frac{3}{2}\right)^{x-2} = \frac{6}{5}\left(\frac{3}{2}\right)^{x-3} - \frac{1}{2}$.

d) $4^{\frac{1}{2}x-1} = 2^{3(x+1)}$.

e) $2^{x^2-6x-\frac{5}{2}} = 16\sqrt{2}$.

f) $2^{2x} + 2^x = 20$.

g) $3^{2x} - 4 \cdot 3^x + 3 = 0$.

h) $\sqrt{2^x} \cdot \sqrt{3^x} = 6^x - 30$

i) $2^x + 3^x = 3^{x+1} - 2^{x+1}$.

Task 5.19 Solve inequalities:

a) $\left(\frac{8}{9}\right)^{8x^2-9} \geq \left(\frac{9}{8}\right)^{9x^2-8}$.

b) $\left(\frac{1}{2}\right)^{2x^2+x-1} > \left(\frac{1}{4}\right)^{\frac{1}{2}x^2+x-\frac{1}{8}}$.

c) $\left(\frac{1}{3}\right)^{|x-3|} \leq \frac{1}{9}$.

d) $5 \cdot 4^x + 2 \cdot 25^x \leq 7 \cdot 10^x$.

e) $2^{x+3} - 5^x < 7 \cdot 2^{x-2} - 3 \cdot 5^{x-1}$.

f) $7^{-x} - 3 \cdot 7^{x+1} > 4$.

Task 5.20 Suppose in 2020 a man purchased a motor yacht Chris Craft Launch 25GT valued at $\$234\,750$. We know that yacht depreciate at 11.2% each year. What would the value of the yacht be in 2026?

Task 5.21 The taxation department allows depreciation of 25% pa on the diminishing value of some kind of computer devices installed on yachts produced by a boatyard. If a boatyard installs computers valued at $\$120\,000$, construct a depreciation schedule for the next five years presenting the information in the table.

Task 5.22 The voltage (V measured in volts) across a capacitor is modelled by the equation $V = 10e^{\frac{-t}{3}}$, where t is measured in seconds. Find V when $t = 5$.

Co-funded by the
Erasmus+ Programme
of the European Union

Task 5.23 The decay of radium is modelled by the function $R = R_0 e^{-0,077t}$, where R is the amount remaining (g), t is time (weeks) and R_0 is the original amount.
Generate a table of values to find the half-life of 10 g of radium.(Remember that half-life means time to reach half of the original amount).

Task 5.24 Carbon dating involves the measurement of concentration of carbon remaining in an object. The decay function $C = 100 \cdot 2^{-0,1786t}$ is used to determine the age of a bone taken from an archaeological dig, where C is the concentration remaining and t is time in thousands of years. It is found that 60% of the original carbon remains in the samples.
Estimate the age of the bone. (Hint: Develop a table of values for the inverse function and find when $C = 60$).

Co-funded by the
Erasmus+ Programme
of the European Union

Answers

5.18.

a) $x = -1$
b) $x = 0$
c) $x = 2$
d) $x = -\frac{5}{2}$
e) $x = -1$ or $x = 7$
f) $x = 2$
g) $x = 0$ or $x = 1$
h) $x = 2$
i) $x = 1.$

5.19.

a) $x \in [-1,\ 1]$
b) $x \in \left(-\frac{1}{2},\ \frac{3}{2}\right)$
c) $x \in (-\infty, 1] \cup [5, \infty)$
d) $x \in [0,\ 1]$
e) $x \in (3, \infty)$
f) $x \in (-\infty, -1]$

5.20. $\$\ 115102.14$

5.21.

Year 1	$90000
Year 2	$67500
Year 3	$50625
Year 4	$37968.75
Year 5	$28476.56

5.22. $V = 1.89.$

5.23.

Original function	
Weeks t	Radium $f(t)$ (g)
0	10,00
1	9,26
2	8,57
3	7,94
4	7,35
5	6,80
6	6,30
7	5,83
8	5,40
9	5,00

From the table we see that 10 grams of radium is reduced to 5 grams in 9 weeks.

5.24.

Original function		Inverse function	
Thousands of years t	Carbon	Carbon	Thousands of years t
0	100	100	0
1	88	88	1
2	78	78	2
3	69	69	3
4	61	61	4
5	54	54	5

If we start with $100\ g$ then 60% will occur when we have $60\ g$. From either table we can see that we have $60\ g$ when the bone is $4\ 000$ years old.

Co-funded by the Erasmus+ Programme of the European Union

Sample chapter exam

1. Solve the equation: $2 \cdot 4^{\sqrt{x}} = \sqrt[4]{2} \cdot 8^{x-1}$.
2. Solve the inequality: $5^x - 20 > 5^{3-x}$.
3. Find all the values of x for which $f(x) > 0$, if $f(x) = \left(\frac{3}{5}\right)^{x^2-x-6} - 1$.
4. There are given functions: $f(x) = 4^{x+1} - 7 \cdot 3^x$ and $g(x) = 3^{x+2} - 5 \cdot 4^x$. Solve the inequality $f(x) \leq g(x)$.
5. Find the domain and a range of a function $f(x) = e - e^x$.
6. The equation $P = 20 \cdot 10^{0,1n}$ can be used to convert any number of decibels (n) to the corresponding number of micropascals (P) used to measure loudness. Show that a 60 decibel sound is 10 times as loud as a 50 decibel sound, and 100 times as loud as a 40 decibel sound.

Co-funded by the
Erasmus+ Programme
of the European Union