Exercises

Task 5.25 Evaluate:

a. $\log_{\sqrt{2}} 16$
b. $\log_2 \frac{1}{8}$
c. $\log_4 0.5$
d. $\log_{\sqrt{2}} 0.25$
e. $\log_{\frac{2}{3}} 2.25$
f. $\log_{\frac{1}{9}} 3\sqrt[3]{3}$
g. $16^{\log_2 3}$.

Task 5.26 Evaluate $\log_{35} 28$ if we know that $\log_{14} 2 = a, \quad \log_{14} 5 = b.$

Task 5.27 Solve the equations:

a. $\ln(5x - e) = 1$
b. $\log_{1.5}\left(2x - \sqrt[3]{1.5}\right) = \frac{1}{3}$
c. $\log_x 3\sqrt{3} = \frac{1}{2}$
d. $\log_{\frac{3}{4}}\left(1 - \frac{x-2}{2x-5}\right) = -1$
e. $\ln(\log_2 x) = 0.$

Task 5.28 Solve the equations:

a. $\log_3\left(x + \sqrt{3}\right) = -\log_3\left(x - \sqrt{3}\right)$
b. $\log_3(5x + 1) - \log_3(x - 1) = 2$
c. $\log_4 x + \log_4(12 - 2x) = 2$
d. $\log(5 - x) + 2\log\sqrt{x - 3} = 0$
e. $\frac{1}{2}\log(2x + 7) + \log\sqrt{7x + 5} = 1 + \log\frac{9}{2}$
f. $\log_3 x + \log_5 x = \frac{\log 15}{\log 3}$
g. $(\log_3 x)^2 = \frac{1}{2}\log_3 x.$

Task 5.29 Solve inequalities

a. $\log(x - 3) - \log(27 - x) \leq -\log 5 - 1$
b. $\log_{\frac{1}{2}}(\log_5 x) \geq 0$
c. $\log_{\frac{1}{3}}(|x| - 1) > -2$
d. $3^{\log_{\frac{1}{5}}(x^2 - 4x - 4)} < 1$
e. $\log_{x^2}(x + 6) \geq 1$

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f. $\log_{\frac{1}{2}} \frac{2x+1}{3x+2} > 3$

Task 5.30 A particular dangerous bacteria culture, that threatens the marine fauna of the Maldives, doubles every 20 minutes and follows the exponential function $N(t) = 200 \cdot 2^{3t}$, where $N(t)$ is the number of bacteria in the culture after t hours. After how many hours will be $1\,000\,000$ bacteria in the culture?

Task 5.31 Rearrange the following formula to make x the subject: $y = 1.4e^{-0.6x} - 3$.

Answers

4.24

a. 8
b. -3
c. $-\frac{1}{2}$
d. -4
e. -2
f. $-\frac{2}{3}$
g. 81

4.25. $\frac{a+1}{b-a+1}$

4.26.

a. $x = \frac{2}{5}e$
b. $x = \sqrt[3]{1.5}$
c. $x = 27$
d. $x = \frac{11}{5}$
e. $x = 2$

4.27.

a. $x = 2$
b. $x = \frac{5}{2}$
c. $x_1 = 2,\ x_2 = 4$
d. $x = 4$
e. $x = 10$
f. $x = 5$
g. $x_1 = 1,\ x_2 = \sqrt{3}$
h.

4.28.

a. $x \in (3, \frac{59}{17}]$
b. $x \in (1, 5]$
c. $x \in (-10, -1) \cup (1, 10)$
d. $x \in (-\infty, -1) \cup (5, \infty)$
e. $x \in [-2, -1) \cup (1, 3]$
f. $x \in \left(-\frac{1}{2}, -\frac{6}{13}\right)$.

4.29. After approximately 4.096 hours there will be $1\,000\,000$ bacteria in the culture.

4.30. $x = -\frac{5}{3}\ln\left(\frac{y+3}{1.4}\right)$.

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Sample chapter exam

1. Prove the following statements:

   a. $\log_{\sqrt{a}} x = 2\log_a x$,

   a. $\log_{\frac{1}{\sqrt{a}}} \sqrt{x} = -\log_a x$,

   c. $\log_{a^4} x^2 = \log_a \sqrt{x}$.

2. Solve the equation: $\log_{\frac{1}{2}}[\log_2(\log_4 x)] = -1.$

3. Solve the inequality: $\log_{\frac{1}{\sqrt{5}}}(6^{x+1} - 36^x) \geq -2.$

4. Find the domain of the function: $f(x) = \log_{x^2-3}(x^2 + 2x - 3).$

5. Draw the graph of each of the following logarithmic functions and analyze each of them completely (i.e., domain, range, zeros, y −intercept, sign, maximal intervals of monotonicity):

   a. $f(x) = \log(-x)$,

   b. $f(x) = -\log(x - 3).$

6. If $y = 3(\mu e)^k$ show that $k = \frac{\ln y - \ln 3}{\ln \mu + 1}$.

7. If $A = P(1 + i)^n$, find n in terms of A, P and i.

8.* Solve the inequality without using a calculator: $\log_{2008}(x^2 - 2007x) \leq 1.$

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