Excercise 1.

Convert from degree to radian measure or vice versa, and fill in the table:

α°	α_{rad}
60°	
	$\frac{3\pi}{2}$
150°	
	3π
450°	
	$\frac{19\pi}{4}$
-130°	
	$-\frac{\pi}{6}$
1000°	

Solution:

α°	α_{rad}
60°	$\frac{\pi}{3}$
270°	$\frac{3\pi}{2}$
150°	$\frac{5\pi}{6}$
540°	3π
450°	$\frac{5\pi}{2}$
855°	$\frac{19\pi}{4}$
-120°	$\frac{-\pi}{3}$
-30°	$-\frac{\pi}{6}$
1000°	$\frac{50\pi}{9}$

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Exercise 2.

Find the coterminal angles and fill in the table:

$\alpha°$	$coterminal(\alpha)$
$390°$	
$450°$	
$1000°$	
$-200°$	
$-1000°$	
$-721°$	

Solution:

$\alpha°$	$coterminal(\alpha)$
$390°$	$30°$
$450°$	$90°$
$1000°$	$280°$
$-200°$	$120°$
$-1000°$	$80°$
$-721°$	$359°$

Exercise 3

Using the following link, fill in the table with approximate values:

https://www.geogebra.org/calculator/tazfcyed

$\alpha°$	α_{rad}	$\sin(\alpha)$	$\cos(\alpha)$
$30°$			
	$\frac{3\pi}{2}$		
		0	1
$135°$			
	$\frac{7\pi}{6}$		

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		$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{2}}{2}$

Solution:

$\alpha°$	α_{rad}	$\sin(\alpha)$	$\cos(\alpha)$
$30°$	$\frac{\pi}{6}$	0.5	0.87
$270°$	$\frac{3\pi}{2}$	-1	0
$0°$	0	0	1
$135°$	$\frac{3\pi}{4}$	0.71	-0.71
$210°$	$\frac{7\pi}{6}$	-0.5	-0.87
$45°$	$\frac{\pi}{4}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{2}}{2}$

Exercise 4.

α	0	$\pi/6$	$\pi/4$	$\pi/3$	$\pi/2$	π
$\sin(\alpha)$	0	½	$\sqrt{2}/2$	$\sqrt{3}/2$	1	0
$\cos(\alpha)$	1	$\sqrt{3}/2$	$\sqrt{2}/2$	$1/2$	0	-1
$\tan(\alpha)$	0	$1/\sqrt{3}$	1	$\sqrt{3}$	$UNDEFINED$	0

Using the trigonometric function table and coterminal angles, calculate the values:

α_{rad}	$coterminal(\alpha)$	$\sin(\alpha)$	$\cos(\alpha)$	$\tan(\alpha)$
$\frac{13\pi}{6}$				
6π				
$\frac{109\pi}{4}$				
$\frac{49\pi}{2}$				
$\frac{43\pi}{3}$				

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Solution:

α_{rad}	$coterminal(\alpha)$	$\sin(\alpha)$	$\cos(\alpha)$	$\tan(\alpha)$
$\frac{13\pi}{6}$	$\frac{\pi}{6}$	$1/2$	$\sqrt{3}/2$	$1/\sqrt{3}$
6π	0	0	1	0
$\frac{109\pi}{4}$	$\frac{\pi}{4}$	$\sqrt{2}/2$	$\sqrt{2}/2$	1
$\frac{49\pi}{2}$	$\frac{\pi}{2}$	1	0	$UNDEFINED$
$\frac{43\pi}{3}$	$\frac{\pi}{3}$	$\sqrt{3}/2$	$1/2$	$\sqrt{3}$

Exercise 5.

Using the following link, inspect the graph and explain.

https://www.geogebra.org/calculator/baxuhgth

a) Explain the property of parameter A by moving the slider left/right and observing the difference between the two functions? (parameter A is called the **amplitude**) Return the slider to the value A=1.

b) Explain the property of parameter C by moving the slider left/right and observing the difference between the two functions?
(parameter C is called the phase) Return the slider to the value C=0.

c) Explain the property of parameter B by moving the slider left/right and observing the difference between the two functions?
(the number $2\pi/B$ is called the **wavelength**)

Solution:

a) Parameter A determines the maximum and minimum value of the sine function. Therefore, it determines the **amplitude** of the sine function. b) Parameter C determines the left/right **shift** of the sine function. In physics, this shift is called the **phase**. c) Parameter B determines the shrinking/stretching of the sine function. If B=2, the function makes 2 sine waves compared to one sine wave made by the regular sine function. Therefore, the greater the parameter B, the **shorter** is the sine wave, so the wavelength shortens. In physics, the shorter wavelength corresponds to a greater frequency. This is very important in optics, acoustics, etc.

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CONNECTIONS AND APPLICATIONS

Exercise 6.

How to read a radar (why trigonometry matters)?

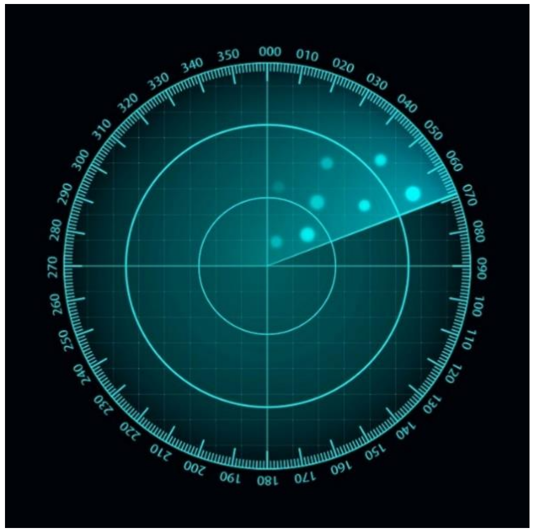

Figure 5.51

The current location of the object is always in the centre of the radar. But what are the funny markings on the side of the radar? In aeronautics, **directions** are determined using the **azimuth.** Azimuth is the angle between the object direction and true north. For example, the azimuth of direction North is 0°, the azimuth of direction West is 270°. Instead of writing angles, the directions on a radar are sets of 3 numbers. For example, the direction of 0° (north) is 000, the direction of East (90°) is 090 (read zero-nine-zero).

What about the circles? Well, all points on the same circle are equally distant from the circle centre, regardless of the direction.

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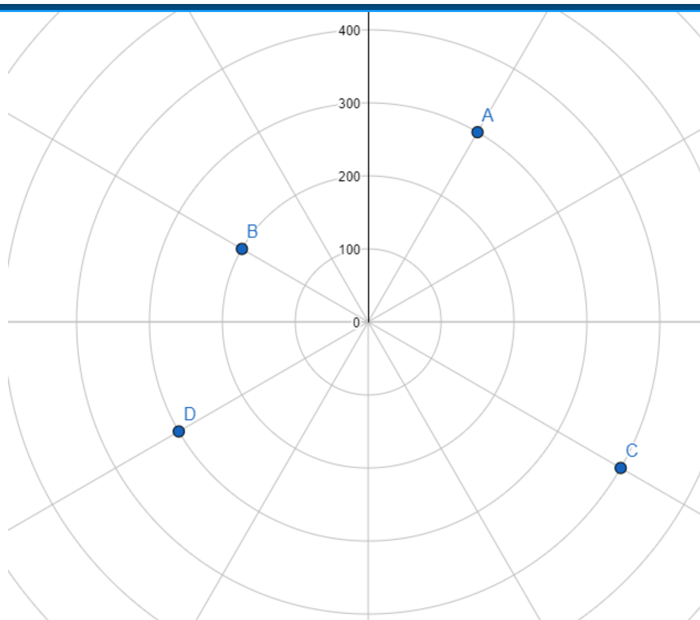

Figure 5.52

The points A and D lie on the same circle, so the distance to the centre is the same.

If a coordinate plane were to be modified so that circles would be drawn from the origin, it would be called a **polar coordinate** system.

Each point in a polar coordinate system has 2 parameters:

1) the distance from the origin (the circle it lies on)

2) the direction (angle) formed by the point and **the positive part of the x axis.**

So, for mathematicians, the direction 0 corresponds to East, the direction North is 90°, the direction West is 180° and the direction South is 270°.

Exercise 7.

A submarine accompanies a ship on its voyage. The position of the submarine and ship are given in the following Geogebra file:

https://www.geogebra.org/calculator/xsmccneq
At each point in time, the ships distance is 200 meters in the direction 060.

a) The submarine captain spots a dangerous reef located directly north from the submarines' location, 200 m away.
   1) Plot the point locating the reef in Geogebra

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2) What is the distance from the ship to the reef?
3) What would be the reefs direction when observed from the ship?

b) The submarine captain spots a potential threat from direction 120, 200 m away from the submarine.
1) Plot the threat location
2) Measure or calculate the threats distance from the ship.

c) The ships safe harbour is located 400 m away, in the direction 330 from the current submarine location.
1) Plot the harbour location
2) Determine the course of the boat if it wants to reach the harbour
3) Determine the distance from the boat to the harbour

Solution:

The complete soliton is shown in the following Geogebra app.

https://www.geogebra.org/m/w7yspzpn

a) To solve this part of the task, plot the reef as a point with coordinates (0,200).
In the app, click on the circle next to the Reef object.
To calculate the distance from the ship, in the menu select distance between points and measure the distance from the ship too the reef.

If you are working with the app, click on the point next to the text object indicating the distance from Ship to Reef.
To calculate the direction of the reef from the ship, draw a line through the points indicating the ship and reef. Then, plot a point directly north from the ship (the x coordinate must remain fixed), and label it North. Lastly, using the Angle tool in Geogebra, measure the angle measure with the first endpoint in point North, the vertex located in the ships position, and the endpoint is the reef.

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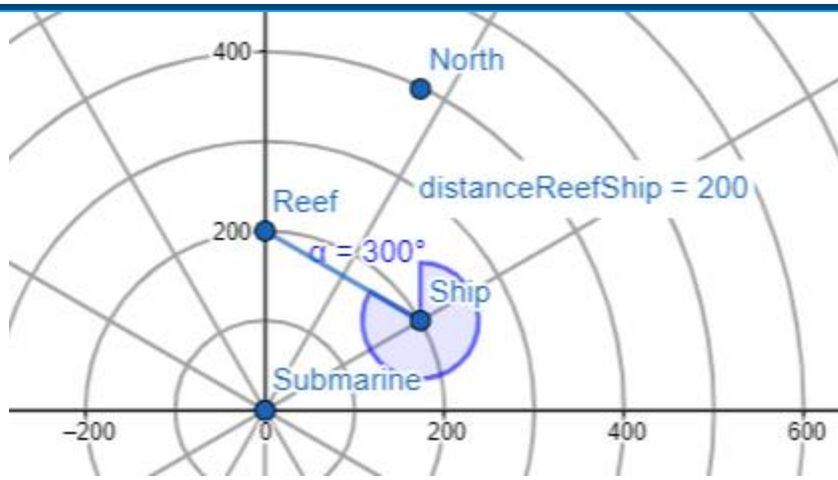

Figure 5.53

If you are using the solution, click on the corresponding objects to make them visible. The objects are the line segment f, the point North and angle α.
The required angle is 300°.

b) To plot the threat location, follow the line corresponding to the angle of 120° (the first line in the lower right quadrant of the coordinate plane. Then, using the circle indicating 200 m, plot the threat as a point in the intersection of the line and the circle, like shown in the picture below.

Lastly, measure the distance between the Submarine and threat using the measuring tool in Geogebra. The solution should look like this:

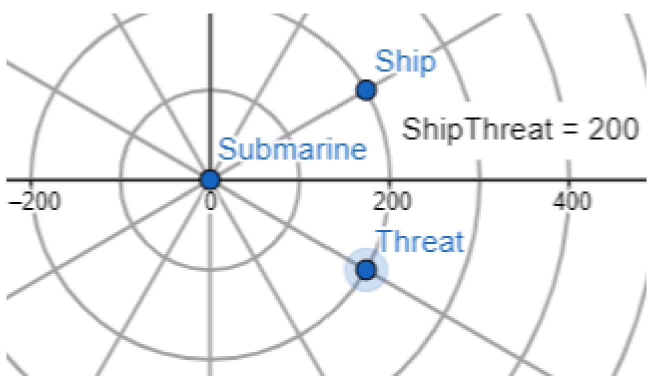

Figure 5.54

If you are using the provided solution, just activate the corresponding objects (point Threat and the text showing the distance)

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c) To indicate the harbour, use the line corresponding to the direction of 330, which is the first line in the upper left quadrant of the coordinate plane. Plot the harbour at the intersection of the line and the circle indicating the distance of 400 m. Using a measuring tool in Geogebra, measure the distance from the ship to the harbour. Using the angle tool in Geogebra, measure the angle from the point North through the vertex Ship and the endpoint Harbour.
The solution is shown below:

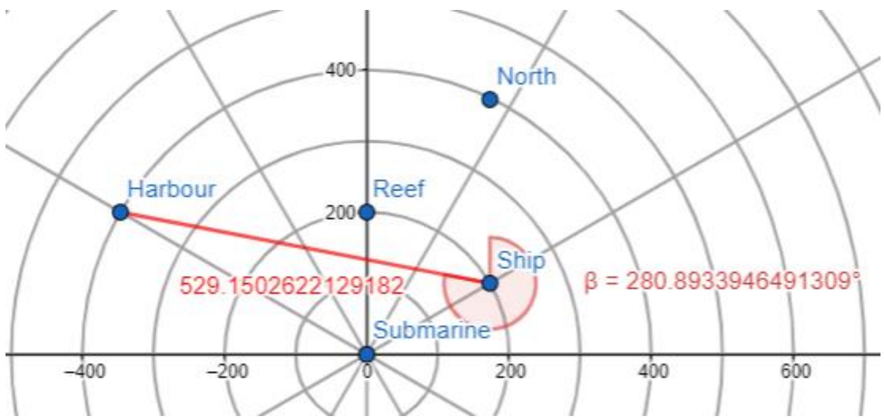

Figure 5.55

If you are using the solution app, just highlight the appropriate objects in the left pane.

Exercise 8.

The following link shows part of the Split sea area, with islands which are close by. https://www.geogebra.org/calculator/bznfax2e

Your ship is located in the Rogac port on the island of Solta.

a) Determine the direction and distance from the Rogac port to Split
b) Determine the direction and distance from the Rogac port to Supetar (look to the west).
c) c A distress signal is sent from point Signal. Using the Geogebra measuring tool, inspect which of the three harbours Split, Rogac or Supetar is the nearest to the location Signal.

Solution:

The complete solution is shown in the following app:

https://www.geogebra.org/calculator/qdbagzsw

a) To determine the direction, first choose a point lying north of port Rogac. The point must have the same x coordinate as port Rogac. To find the distance from Rogac to Split, use the measuring tool provided in Geogebra. To find the direction, use the angle tool in Geogebra,

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with the start point being Split, the vertex being Rogac and the endpoint being North. The final solution should look like this:

b)

Figure 5.56

If the solution app is used, click on the corresponding elements in the left pane to hide/show them on the map. The elements for the first part of the solution are point North, distance from Rogac to Split, and angle α. The angle of 44° corresponds to a direction of 044.

c) The direction of the Supetar port is almost 090 since it is west of port Rogac. To find the exact direction, plot the port Supetar with a point, using the point Supetar as the starting point, the point Rogac as the vertex, and the point North as the endpoint, draw an angle using the angle tool in Geogebra. The angle is 92.7°, so the direction is approximately 093. To determine the distance, draw a line segment from port Rogac to port Supetar, and measure its distance. The distance from Rogac to Supetar should be approximately 19.5 km. Another way to approximate the point Supetar is to use the radar circles. The point Supetar is between the circle indicating the distance of 18 km and the circle indicating the distance of 20 km, so the true distance must be between 18 and 20 km.

The final solution should look like this:

Figure 5.57

If you are using the solution app, just highlight the point Supetar, the line segment with the distance, and angle β.

d) To find the shortest route from the port to point Signal, make a line segment and measure the distance from the port to Signal for all three ports using the Geogebra measuring tool. The solution should look like this:

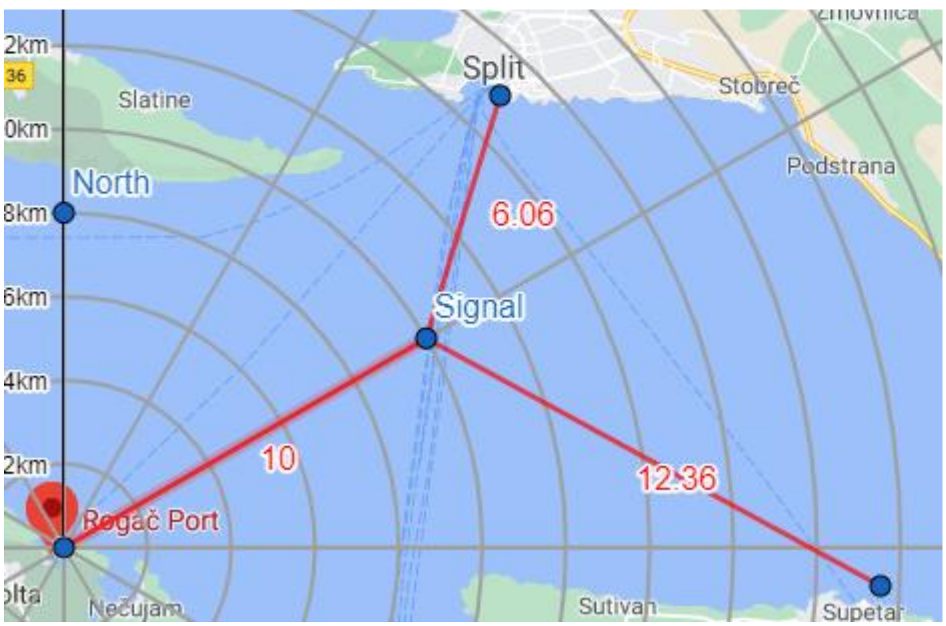

Figure 5.58

So, the nearest port is Split, so the rescue ship should be sent from Split. If you are using the solution app, show the corresponding elements.

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