5.7.5. APPLICATION EXAMPLES

Example 5.62 ARCHITECTURE

Figure 5.83

The support for a roof is shaped like two right triangles, as shown below. Find θ

Solution:

Figure 5.80 shows a right triangle with

$$\sin\theta = \frac{side\ opposite\ \theta}{hypotenuse} = \frac{9}{18} = \frac{1}{2}.$$

When $\sin\theta = \frac{1}{2}$. We must find the angle $\theta\epsilon\left[-\frac{\pi}{2};\frac{\pi}{2}\right]$, whose sine equals $\frac{1}{2}$. Use the exact values in Table 1 to find the value of θ in $\left[-\frac{\pi}{2};\frac{\pi}{2}\right]$ that satisfies $\sin\theta = \frac{1}{2}$. Table 1 shows that the only angle in the interval $\left[-\frac{\pi}{2};\frac{\pi}{2}\right]$ that satisfies $\sin\theta = \frac{1}{2}$ is $\frac{\pi}{6}$. Thus, $\theta = \frac{\pi}{6}$.

Example 5.63 RESCUE

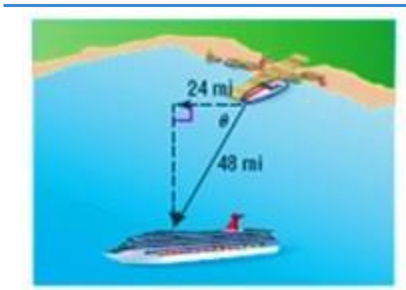

Figure 5.84

A cruise ship sailed due west 24 miles before turning south. When the cruise ship became disabled and the crew radioed for help, the rescue boat found that the fastest route covered a distance of 48 miles. Find the angle θ at which the rescue boat should travel to aid the cruise ship.

Solution:

Figure 16 shows a right triangle with

$$\cos\theta = \frac{side\ adjacent\ \text{to}\ \theta}{hypotenuse} = \frac{24}{48} = \frac{1}{2}.$$

When $\cos\theta = \frac{1}{2}$. We must find the angle $\theta\epsilon[0;\pi]$, whose cosine equals $\frac{1}{2}$. Use the exact values in Table 2 to find the value of θ in $[0;\pi]$ that satisfies $\cos\theta = \frac{1}{2}$. Table 2 shows that the only angle in the interval $[0;\pi]$ that satisfies $\cos\theta = \frac{1}{2}$ is $\frac{\pi}{3}$. Thus, $\theta = \frac{\pi}{3}$.

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Example 5.64 *DRAG RACE*

Figure 5.85

A television camera is filming a drag race. The camera rotates as the vehicles move past it. The camera is 30 meters away from the track. Consider θ and x as shown in the ***Figure 5.82.***

a. Find θ when $x = 6$ meters and $x = 14$ meters.

b. Write θ as a function of x.

Solution:

a. Figure 17 shows a right triangle with the relationship between θ and the sides is opposite and adjacent, so

$$\tan\theta = \frac{side\ opposite\ \theta}{side\ adjacent\ \text{to}\ \theta} = \frac{x}{30}.$$

If $x = 6$, then $\tan\theta = \frac{x}{30} = \frac{6}{30} = \frac{1}{5}$. We must find the angle $\theta \epsilon \left(-\frac{\pi}{2}; \frac{\pi}{2}\right)$, whose tangent equals $\frac{1}{5}$. We cannot use the exact values in Table 3 to find the value of θ in $\left(-\frac{\pi}{2}; \frac{\pi}{2}\right)$ that satisfies $\tan\theta = \frac{1}{5}$, because we have not such value in this table. But we can use a calculator -> $\tan^{-1}\frac{1}{5} \approx 11.3°$ This angle is in the interval $\left(-\frac{\pi}{2}; \frac{\pi}{2}\right)$ and satisfies $\tan\theta = \frac{1}{5}$.

If $x = 14$, then $\tan\theta = \frac{x}{30} = \frac{14}{30} = \frac{7}{15}$. We must find the angle $\theta \epsilon \left(-\frac{\pi}{2}; \frac{\pi}{2}\right)$, whose tangent equals $\frac{7}{15}$. We cannot use the exact values in Table 3 to find the value of θ in $\left(-\frac{\pi}{2}; \frac{\pi}{2}\right)$ that satisfies $\tan\theta = \frac{7}{15}$, because we have not such value in this table. But we can use a calculator -> $\tan^{-1}\frac{7}{15} \approx 25°$ This angle is in the interval $\left(-\frac{\pi}{2}; \frac{\pi}{2}\right)$ and satisfies $\tan\theta = \frac{7}{15}$.

b. In step 1 we find

$$\tan\theta = \frac{x}{30}$$

$$\textit{take}\ \arctan\ \textit{of both sides}$$

$$\arctan(\tan\theta) = \arctan\frac{x}{30}$$

$$\textit{use inverse tangent proerties}\ \arctan(\tan x) = x, \text{ for every } x \in \left(-\frac{\pi}{2}; \frac{\pi}{2}\right)$$

Thus, θ as a function of x is

$$\theta = \arctan\frac{x}{30}.$$

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