5.8.2. Exercises

Task 5.38 Find the limits

1. $\lim\limits_{x\to 3}(x^2-2)$
2. $\lim\limits_{x\to \infty} x^2$
3. $\lim\limits_{x\to 0}\frac{1}{x^2}$
4. $\lim\limits_{x\to \infty}\frac{2}{x^3}$
5. $\lim\limits_{x\to \infty}\frac{x-6}{2x+1}$
6. $\lim\limits_{x\to \infty}\frac{3x^2+2x-6}{4x^2-5}$
7. $\lim\limits_{x\to \infty}\frac{5x^3+2}{3x^2+x-3}$
8. $\lim\limits_{x\to 1}\frac{x-1}{x^2-1}$

Solutions

1. $\lim\limits_{x\to 3}(x^2-2)=(3^2-2)=7$

   This exercise was simple. We can just substitute 3 instead of x and calculate. Property P1 is used in this example

2. $\lim\limits_{x\to \infty} x^2=[\infty^2]=\infty$

   If x approaches infinity, we can assume that x is a huge number, e.g. 1000 or even greater and substitute 1000 instead of x in our expression. Since $1\,000^2=1\,000\,000$ is even higher, we can conclude that the result is infinity.

   Notice that we put ∞^2 in brackets $[\infty^2]$. That is because ∞^2 is not a regular mathematical expression.

3. $\lim\limits_{x\to 0}\frac{1}{x^2}=\left[\frac{1}{0^2}\right]=\left[\frac{1}{0}\right]=\infty$

   Dividing by zero is not defined, so we can imagine that x is a very small number, e.g. x=0,001 or even smaller. In that case the value of expression $\frac{1}{x^2}=\frac{1}{0{,}001^2}=\frac{1}{0{,}000001}=1\,000\,000$ which is pretty large number. We can conclude that as the x approaches to zero, the value of expression gets greater and approaches to infinity.

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4. $\lim_{x\to\infty} \frac{2}{x^3} = \left[\frac{2}{\infty}\right] = 0$

When we divide some number (e.g. 2) with some big number, e.g. 1000^3 or even greater the result is a very small number, close to zero.

5. $\lim_{x\to\infty} \frac{x-6}{2x+1} = \left[\frac{\infty}{\infty}\right]$

As x gets greater, the values of denominator and numerator are approaching to infinity. If denominator and numerator are polynomials we can solve this problem by dividing all through by the highest power of x. In this exercise we divided by x.

$$\lim_{x\to\infty} \frac{x-6}{2x+1} = \left[\frac{\infty}{\infty}\right] = \lim_{x\to\infty} \frac{1-\frac{6}{x}}{2+\frac{1}{x}}$$

Since $\lim_{x\to\infty} \frac{6}{x} = 0$ and $\lim_{x\to\infty} \frac{1}{x} = 0$ we get

$$\lim_{x\to\infty} \frac{x-6}{2x+1} = \left[\frac{\infty}{\infty}\right] = \lim_{x\to\infty} \frac{1-\frac{6}{x}}{2+\frac{1}{x}} = \frac{1}{2}$$

6. $\lim_{x\to\infty} \frac{3x^2+2x-6}{4x^2-5} = \left[\frac{\infty}{\infty}\right] = \lim_{x\to\infty} \frac{\frac{3x^2}{x^2}+\frac{2x}{x^2}-\frac{6}{x^2}}{\frac{4x^2}{x^2}-\frac{5}{x^2}} = \lim_{x\to\infty} \frac{3+\frac{2}{x}-\frac{6}{x^2}}{4-\frac{5}{x^2}} = \frac{3}{4}$

Since both denominator and numerator are polynomials, we divided all through by the x^2.

7. $\lim_{x\to\infty} \frac{5x^3+2}{3x^2+x-3} = \left[\frac{\infty}{\infty}\right] = \lim_{x\to\infty} \frac{5+\frac{2}{x^3}}{\frac{3}{x}+\frac{1}{x^2}-\frac{3}{x^3}} = \left[\frac{5}{0}\right] = \infty$

8. $\lim_{x\to 1} \frac{x-1}{x^2-1} = \left[\frac{0}{0}\right]$

If we divide denominator and numerator by x^2 it will not be helpful.

In cases when the values of denominator and numerator both approach to zero, we can solve the problem by using algebra to simplify the expression.

We know that $x^2 - 1 = (x-1)(x+1)$

$$\lim_{x\to 1} \frac{x-1}{x^2-1} = \left[\frac{0}{0}\right] = \lim_{x\to 1} \frac{x-1}{(x-1)(x+1)} = \lim_{x\to 1} \frac{1}{x+1} = \frac{1}{2}$$

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