2.13. EXERCISES

Task 2.1

Determine the type of the default matrix and the required element:

1. $B=\begin{bmatrix}55 & 44\end{bmatrix};\ b_{12}$

2. $C=\begin{bmatrix}48 & 18\\ 6 & 0\\ -6 & 5\\ 18 & -15\end{bmatrix};\ c_{31}$

3. $A=\begin{bmatrix}v_{11} & v_{12} & v_{13} & \cdots & v_{1q}\\ v_{21} & v_{22} & v_{23} & \cdots & v_{2q}\\ \vdots & \vdots & \vdots & \ddots & \vdots\\ v_{p1} & v_{p2} & v_{p3} & \cdots & v_{pq}\end{bmatrix},\ a_{22}$

4. $E=\begin{bmatrix}d & d & d & d & d\end{bmatrix};\ e_{1r}\ \left(\text{za svaki } r\in\{1,2,\ldots,5\}\right)$

Task 2.2

Find the values of $x,\ y,\ z$ and w from the following equation

$$\begin{bmatrix}x-y & x-z\\ y-w & w\end{bmatrix}=\begin{bmatrix}0 & 0\\ 0 & 6\end{bmatrix}.$$

Task 2.3

1. If matrices are $A=\begin{bmatrix}-1 & 1 & 0\\ 0 & -2 & 1\end{bmatrix}$, $B=\begin{bmatrix}3 & 0 & -1\\ 5 & -1 & 1\end{bmatrix}$ and $C=\begin{bmatrix}1 & x & w\\ r & z & 4\end{bmatrix}$. Determine the type of the matrix:

a) $A+B,$ b) $B-C,$ c) $A-B+C,$ d) $\frac{1}{2}B,$

e) $2A-B,$ f) $2A-4C,$ g) $3B^T,$ h) $2A^T-C^T.$

Task 2.4

$A=\begin{bmatrix}44.2 & 0 & 12.2\\ 1.5 & -2.35 & 5.6\end{bmatrix}$, $B=\begin{bmatrix}5.4 & 0\\ 1.4 & 7.8\\ 5.6 & 6.6\end{bmatrix}$ and $C=\begin{bmatrix}-10 & -20 & -30\\ 10 & 20 & 30\end{bmatrix}$. Calculate:

a) $A-C,$ b) $C-A,$ c) $1.1B,$ d) $-0.2B,$

e) $A^T+4.2B,$ f) $\left(A+2.3C\right)^T,$ g) $\left(2.1A-2.3C\right)^T,$ h) $\left(A-C\right)^T-B.$

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Task 2.5

Determine the matrix $F = AB$ if

a) $A = \begin{bmatrix} 0 & -1 & 1 \\ 3 & -1 & 1 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & 1 \\ 0 & 1 \\ 4 & 2 \end{bmatrix}$,

b) $A = \begin{bmatrix} 1 & 3 & 2 \end{bmatrix}$ and $B = \begin{bmatrix} -1 \\ 1 \end{bmatrix}$,

c) $A = \begin{bmatrix} 1.2 & 1.3 & 1.1 & 1.1 \\ 3.4 & 4.4 & 2.3 & 1.1 \\ 2.3 & 0 & -2.2 & 1.1 \\ 1.1 & 2.3 & 3.4 & -1.2 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & 2.2 & 9.8 \\ -3.4 & -4.8 & -4.2 \\ 3.4 & 5.6 & 1 \\ -2.1 & 0 & -3.3 \end{bmatrix}$,

d) $A = \begin{bmatrix} 1 & -7 & 0 & 1 \\ 0 & 2 & 4 & -1 \\ 0 & -2 & 1 & -1 \\ 1 & 1 & -7 & 0 \end{bmatrix}$ and $B = \begin{bmatrix} 1 \\ 3 \\ 2 \\ -1 \end{bmatrix}$.

Task 2.6

If the matrix A $A = \begin{bmatrix} 1 & 1 \\ 1 & 3 \end{bmatrix}$ and $P_4(x) = -x^4 + 3x^3 + 2x^2 - 2x$, determine $P_4(A)$.

Task 2.7

If the matrix B is $B = \begin{bmatrix} -4 & -4 & -1 \\ 3 & 3 & 1 \\ -4 & -4 & 0 \end{bmatrix}$. Calculate $Q_3(B)$ if $Q_3(x) = 4x^3 + 3x^2 - 2x - 1$.

Task 2.8 Calculate:

a) $\begin{vmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 0 & 0 & 0 \end{vmatrix}$, b) $\begin{vmatrix} 1 & 3 & -5 & -7 \\ 3 & -5 & -2 & -1 \\ 1 & 3 & 0 & -2 \\ 3 & -5 & 1 & 4 \end{vmatrix}$, c) $\begin{vmatrix} 1 & 3 & 1 & 6 \\ 1 & 2 & 0 & 7 \\ -1 & 3 & -15 & 18 \\ 2 & -1 & 5 & 6 \end{vmatrix}$,

d) $\begin{vmatrix} 0 & -c & b & -x \\ c & 0 & -a & -y \\ -b & a & 0 & -z \\ x & y & z & 0 \end{vmatrix}$.

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Task 2.9 $n \in \mathbb{R}$

a) $A = \begin{bmatrix} 1 & 2 & 3 & \cdots & n \\ 0 & 1 & 2 & \cdots & n-1 \\ 0 & 0 & 1 & \cdots & n-2 \\ . & . & . & . & . \\ 0 & 0 & 0 & \cdots & 1 \end{bmatrix}$,

b) $B = \begin{bmatrix} 1 & n & n & \cdots & n \\ n & 2 & n & \cdots & n \\ n & n & 3 & \cdots & n \\ . & . & . & . & . \\ n & n & n & \cdots & n \end{bmatrix}$.

Task 2.10 Determine all $x \in \mathbb{R}$ for which

$$A = \begin{bmatrix} 2 & 1 & 3 & 1 \\ 2 & 2-x^2 & 3 & 1 \\ 1 & 3 & 5 & 2 \\ 1 & 3 & 9-x^2 & 2 \end{bmatrix}$$

is singular.

Task 2.11 Find A^{-1} if

a) $A = \begin{bmatrix} 5 & 9 \\ 3 & 4 \end{bmatrix}$,

b) $A = \begin{bmatrix} -1 & -2 \\ -3 & -4 \end{bmatrix}$,

c) $A = \begin{bmatrix} 5 & 1 & -2 \\ -2 & 0 & 5 \\ 7 & 2 & 8 \end{bmatrix}$,

d) $A = \begin{bmatrix} 3 & 1 & 10 \\ 2 & 5 & 4 \\ -1 & -3 & 1 \end{bmatrix}$,

e) $A = \begin{bmatrix} 5 & -5 & 5 & 6 \\ 2 & 1 & 1 & 2 \\ -1 & -1 & 0 & 1 \\ 5 & 1 & 2 & 1 \end{bmatrix}$,

f) $A = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & 1 & -1 & -1 \\ 1 & -1 & 1 & -1 \\ 1 & -1 & -1 & 1 \end{bmatrix}$.

Task 2.12 Solve the matrix equation $(AXB)^{-1} = (X^{-1} + A)A^{-1}$

$A = \begin{bmatrix} 2 & -3 & 1 \\ 3 & -5 & -1 \\ 3 & -4 & 5 \end{bmatrix}$, $B = \begin{bmatrix} 2 & 3 & 1 \\ -2 & 1 & 2 \\ 1 & -2 & 2 \end{bmatrix}$.

Task 2.13

Solve the matrix equation $(XA + C)(AX + 2AB)^{-1} = A^{-1}$ if

$A = \begin{bmatrix} -1 & 2 & 3 \\ 0 & 3 & -1 \\ 0 & 0 & 2 \end{bmatrix}$, $B = \begin{bmatrix} 2 & -1 & 1 \\ 0 & -1 & 1 \\ 0 & 0 & 1 \end{bmatrix}$ i $C = \begin{bmatrix} 1 & -1 & 2 \\ -1 & 0 & 2 \\ 0 & 0 & 2 \end{bmatrix}$.

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Task 2.14 Solve the equation:

$$\begin{bmatrix} 2 & -3 & 1 \\ 4 & -5 & 2 \\ 5 & -7 & 3 \end{bmatrix} \cdot X \cdot \begin{bmatrix} 9 & 7 & 6 \\ 1 & 1 & 2 \\ 1 & 1 & 1 \end{bmatrix} = \begin{bmatrix} 2 & 0 & -2 \\ 18 & 12 & 9 \\ 23 & 15 & 11 \end{bmatrix}.$$

Task 2.15 Solve the matrix equation $AX + 2B = C + BX$,

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 0 & 2 & 4 \\ 0 & 0 & 1 \end{bmatrix}, B = \begin{bmatrix} -1 & 2 & -3 \\ 0 & 4 & 2 \\ 0 & 0 & 2 \end{bmatrix}, C = \begin{bmatrix} 3 & 0 & 1 \\ 0 & 2 & 4 \\ 0 & 0 & 1 \end{bmatrix}.$$

Task 2.16 Solve the matrix equation $AXB^{-1} = I - A$,

$$A = \begin{bmatrix} 3 & 3 & 2 \\ -4 & 1 & -4 \\ -3 & 1 & -3 \end{bmatrix}, \quad B = \begin{bmatrix} 1 & 0 & 2 \\ 1 & -2 & 0 \\ 0 & -1 & 1 \end{bmatrix}.$$

Task 2.17 Solve the matrix equation $(A+3I)(X-I) = B, \quad I$

$$A = \begin{bmatrix} -2 & 5 & -2 \\ 2 & 8 & 0 \\ -1 & -5 & -2 \end{bmatrix} \text{ and } B = \begin{bmatrix} -3 & 21 & 1 \\ 2 & 50 & -2 \\ 1 & -22 & 0 \end{bmatrix}.$$

Task 2.18 Determine the rank of the following matrices:

a) $A = \begin{bmatrix} 0 & 0 & -3 & 8 & -2 \\ -3 & 6 & 12 & 16 & -34 \\ 1 & -2 & -9 & 8 & 8 \end{bmatrix}$,

b) $B = \begin{bmatrix} 3 & -7 & -1 & 3 & 2 \\ 1 & -6 & 1 & 7 & 7 \\ 9 & 2 & -11 & -1 & -7 \\ -5 & -4 & 7 & -7 & -3 \end{bmatrix}$,

c) $C = \begin{bmatrix} 1 & 1 & 1 & 0 & -2 & 0 \\ -2 & -1 & -1 & -1 & 1 & -1 \\ -3 & -2 & -2 & -1 & 3 & -1 \\ -8 & -2 & -2 & -6 & -2 & -6 \\ 3 & -1 & -1 & 4 & 6 & 4 \end{bmatrix}$,

d) $D = \begin{bmatrix} -1 & -1 & 1 & -1 & 1 & 1 \\ -1 & -1 & -1 & 1 & 0 & 1 \\ 0 & -1 & 0 & 0 & 0 & -1 \\ 1 & -1 & 1 & 0 & 0 & -1 \\ -1 & -1 & 0 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 & 1 & 0 \end{bmatrix}$.

Task 2.19 Solve the system using Cramer's rule and

$$\begin{aligned} 3x_1 + x_2 + 3x_3 &= 2 \\ -2x_1 + 2x_2 - 2x_3 &= 2 \\ 3x_2 + 3x_3 &= 2 \end{aligned}$$

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Task 2.20 Can the system

$$\begin{aligned} 4x_1 &- 3x_2 = 8 \\ 4x_1 &- x_2 = 4 \\ 2x_1 &+ x_2 = -1 \\ &- x_2 = 2 \end{aligned}$$

be solved using Cramer's rule?

Note: The rank is determined first. The result is $n = r(A) = r(\tilde{A}) = 2$ and a new system (equivalent to the default) with a system matrix C and an extended matrix system of the $\tilde{C}$ u in which $m = n = r(C) = r(\tilde{C}) = 2,$ this system can be solved using Cramer's rule.

Task 2.21 Calculate all system solutions

$$\begin{aligned} 6x_1 &+ x_2 + 3x_3 - 4x_4 - 4x_5 = 3 \\ -2x_1 &+ x_2 + x_3 - 2x_4 - 2x_5 = 1 \\ 3x_1 &- x_2 + x_3 = 0 \\ 3x_1 &- x_3 + x_4 + x_5 = 0 \end{aligned}$$

Is vector $X^* = \frac{1}{10}\begin{bmatrix} 11 \\ 100 \\ 0 \\ 34 \\ 0 \end{bmatrix}$ solution to the system?

Task 2.22 How many solutions does the system have?

$$\begin{aligned} 2x_1 &- x_2 - x_3 + 2x_4 + x_5 = 1 \\ 6x_1 &+ x_2 - 5x_3 + 9x_4 - 3x_5 = -2 \\ 2x_1 &+ x_2 + 3x_3 + 3x_4 + x_5 = 0 \\ &- x_2 + 3x_3 - x_4 + 3x_5 = 2 \\ &- 4x_2 - 8x_3 - 2x_4 = 1 \end{aligned} \quad ?$$

Task 2.23 Calculate all system solutions.

$$\begin{aligned} x_1 &+ x_2 + x_3 - 2x_5 = 0 \\ 2x_1 &+ x_2 + x_3 + x_4 - x_5 = 1 \\ 3x_1 &+ 2x_2 + 2x_3 + x_4 - 3x_5 = 1 \\ 4x_1 &+ x_2 + x_3 + 3x_4 + x_5 = 3 \\ 3x_1 &- x_2 - x_3 + 4x_4 + 6x_5 = 4 \end{aligned}$$

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Task 2.24 For which $p \in \mathbb{R}$ is system

$$\begin{aligned} -2x_1 &+ x_2 - 2x_3 = 3 \\ 2x_1 &+ 2x_2 + x_3 = -2 \\ 4x_1 &+ px_2 + 4x_3 = -7 \end{aligned}$$

impossible?

Is there $p \in \mathbb{R}$ for which a system has infinite solutions?

What is the value of $p \in$ for vector $X = \begin{bmatrix} 1/3 \\ -1/3 \\ -2 \end{bmatrix}$ to be the solution to that system?

Answers:

Task 2.1

1. B is type 1×2; $b_{12} = 44.$
2. C is type 4×2; $c_{31} = -6.$
3. A is type $p\times q$; $a_{22} = v_{22}.$
4. E is type 1×5; $e_{1r} = d.$

Task 2.2 $x = y = z = w = 6.$

Task 2.3

a) $A+B = \begin{bmatrix} 2 & 1 & -1 \\ 5 & -3 & 2 \end{bmatrix}$,

b) $B-C = \begin{bmatrix} 2 & -x & -1-w \\ 5-r & -1-z & -3 \end{bmatrix}$,

c) $A-B+C = \begin{bmatrix} -3 & 1+x & 1+w \\ -5+r & -1+z & 4 \end{bmatrix}$,

d) $\frac{1}{2}B = \begin{bmatrix} 1.5 & 0 & -0.5 \\ 2.5 & -0.5 & 0.5 \end{bmatrix}$,

e) $2A-B = \begin{bmatrix} -5 & 2 & 1 \\ -5 & -3 & 1 \end{bmatrix}$,

f) $2A-4C = \begin{bmatrix} -6 & 2-4x & -4w \\ -4r & -4-4z & -14 \end{bmatrix}$,

g) $3B^T = \begin{bmatrix} 9 & 15 \\ 0 & -3 \\ -3 & 3 \end{bmatrix}$,

h) $2A^T - C^T = \begin{bmatrix} -3 & -r \\ 2-x & -4-z \\ -w & -2 \end{bmatrix}$.

Task 2.4

a) $A-C = \begin{bmatrix} 54.2 & 20 & 42.2 \\ -8.5 & -22.35 & -24.4 \end{bmatrix}$,

b) $C-A = \begin{bmatrix} -54.2 & -20 & -42.2 \\ 8.5 & 22.35 & 24.4 \end{bmatrix}$,

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c) $1.1B=\begin{bmatrix}5.94 & 0\\ 1.54 & 8.58\\ 6.16 & 7.26\end{bmatrix}$, d) $-0.2B=\begin{bmatrix}-1.08 & 0\\ -0.28 & -1.56\\ -1.12 & -1.32\end{bmatrix}$,

e) $A^T+4.2B=\begin{bmatrix}66.88 & 1.5\\ 5.88 & 30.41\\ 35.72 & 33.32\end{bmatrix}$, f) $\left(A+2.3C\right)^T=\begin{bmatrix}21.2 & 24.5\\ -46 & 43.65\\ -56.8 & 74.6\end{bmatrix}$,

g) $\left(2.1A-2.3C\right)^T=\begin{bmatrix}115.82 & -19.85\\ 46 & -50.94\\ 94.62 & -57.24\end{bmatrix}$, h) $\left(A-C\right)^T-B=\begin{bmatrix}48.8 & -8.5\\ 18.6 & -30.15\\ 36.6 & -31\end{bmatrix}$.

Task 2.5

a) $F=\begin{bmatrix}4 & 1\\ 7 & 4\end{bmatrix}$, b) F does not exist,

c) $F=\begin{bmatrix}-1.79 & 2.56 & 3.77\\ -6.05 & -0.76 & 13.51\\ -7.49 & -7.26 & 16.71\\ 7.36 & 10.42 & 8.48\end{bmatrix}$, d) $F=\begin{bmatrix}-21\\ 15\\ -3\\ -10\end{bmatrix}$.

Task 2.6 $P_4\left(A\right)=\begin{bmatrix}0 & 0\\ 0 & 0\end{bmatrix}$.

Task 2.7 $Q_3\left(B\right)=\begin{bmatrix}-1 & 0 & 2\\ 1 & 0 & -2\\ 4 & 4 & -1\end{bmatrix}$.

Task 2.8 a) 0, b) -140, c) 40, d) $\left(ax+by+cz\right)^2$.

Task 2.9 Instruction: Apply the property of determinant 5).

a) $\det A=1$,

b)

$$\det B=\begin{vmatrix}1 & n & n & \cdots & n & n\\ n & 2 & n & \cdots & n & n\\ n & n & 3 & \cdots & n & n\\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot\\ n & n & n & \cdots & n-1 & n\\ n & n & n & \cdots & n & n\end{vmatrix}\begin{matrix}R_1-R_n\\ R_2-R_n\\ R_3-R_n\\ \vdots\\ R_{n-1}-R_n\\ \end{matrix}=\begin{vmatrix}1-n & 0 & 0 & \cdots & 0 & 0\\ 0 & 2-n & 0 & \cdots & 0 & 0\\ 0 & 0 & 3-n & \cdots & 0 & 0\\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot\\ 0 & 0 & 0 & \cdots & (n-1)-n & 0\\ n & n & n & \cdots & n & n\end{vmatrix}=$$

$$=\left(1-n\right)\left(2-n\right)\cdots\left(\left(n-1\right)-n\right)n=\left(-1\right)^{n-1}n!$$

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Task 2.10 Matrix A is singular if $x \in \{-2,-1,1,2\}$.

Task 2.11

a) $A^{-1} = \frac{1}{7}\begin{bmatrix} -4 & 9 \\ 3 & -5 \end{bmatrix}$, b) $A^{-1} = \frac{1}{2}\begin{bmatrix} 4 & -2 \\ -3 & 1 \end{bmatrix}$,

c) $A^{-1} = \frac{1}{9}\begin{bmatrix} -10 & -12 & 5 \\ 51 & 54 & -21 \\ -4 & -3 & 2 \end{bmatrix}$, d) $A^{-1} = \frac{1}{35}\begin{bmatrix} 17 & -31 & -46 \\ -6 & 13 & 8 \\ -1 & 8 & 13 \end{bmatrix}$,

e) $A^{-1} = \frac{1}{8}\begin{bmatrix} -4 & -8 & 26 & 14 \\ 2 & 8 & -19 & -9 \\ 10 & 16 & -63 & -29 \\ -2 & 0 & 15 & 5 \end{bmatrix}$, f) $A^{-1} = \frac{1}{4}\begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & 1 & -1 & -1 \\ 1 & -1 & 1 & -1 \\ 1 & -1 & -1 & 1 \end{bmatrix}$.

Task 2.11

$$(AXB)^{-1} = \left(X^{-1} + A\right)A^{-1}$$
$$B^{-1}X^{-1}A^{-1} = \left(X^{-1} + A\right)A^{-1} \,/\cdot A$$
$$B^{-1}X^{-1} = X^{-1} + A \,/\cdot X$$
$$B^{-1} = I + AX$$
$$A^{-1}\cdot /B^{-1} - I = AX$$
$$A^{-1}\left(B^{-1} - I\right) = X;$$

$$A^{-1} = \begin{bmatrix} 29 & -11 & -8 \\ 18 & -7 & -5 \\ -3 & 1 & 1 \end{bmatrix} \qquad B^{-1} = \frac{1}{33}\begin{bmatrix} 6 & -8 & 5 \\ 6 & 3 & -6 \\ 3 & 7 & 8 \end{bmatrix}$$

$$B^{-1} - I = \frac{1}{33}\begin{bmatrix} 6 & -8 & 5 \\ 6 & 3 & -6 \\ 3 & 7 & 8 \end{bmatrix} - \frac{1}{33}\begin{bmatrix} 33 & 0 & 0 \\ 0 & 33 & 0 \\ 0 & 0 & 33 \end{bmatrix} = \frac{1}{33}\begin{bmatrix} -27 & -8 & 5 \\ 6 & -30 & -6 \\ 3 & 7 & -25 \end{bmatrix}$$

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$$X = A^{-1}\left(B^{-1} - I\right) = \begin{bmatrix} 29 & -11 & -8 \\ 18 & -7 & -5 \\ -3 & 1 & 1 \end{bmatrix} \frac{1}{33} \begin{bmatrix} -27 & -8 & 5 \\ 6 & -30 & -6 \\ 3 & 7 & -25 \end{bmatrix} = \frac{1}{33} \begin{bmatrix} -873 & 42 & 411 \\ -543 & 31 & 257 \\ 90 & 1 & -46 \end{bmatrix} =$$

$$= \begin{bmatrix} -\frac{291}{11} & \frac{14}{11} & \frac{137}{11} \\ -\frac{181}{11} & \frac{31}{33} & \frac{257}{33} \\ \frac{30}{11} & \frac{1}{33} & -\frac{46}{33} \end{bmatrix}.$$

Task 2.13

$$\left(XA + C\right)\left(AX + 2AB\right)^{-1} = A^{-1} \,/\cdot \left(AX + 2AB\right)$$
$$XA + C = A^{-1}\left(AX + 2AB\right)$$
$$XA + C = X + 2B$$
$$XA - X = 2B - C$$
$$X\left(A - I\right) = 2B - C \,/\cdot \left(A - I\right)^{-1}$$
$$X = \left(2B - C\right)\left(A - I\right)^{-1};$$

$$2B = \begin{bmatrix} 4 & -2 & 2 \\ 0 & -2 & 2 \\ 0 & 0 & 2 \end{bmatrix} \qquad 2B - C = \begin{bmatrix} 3 & -1 & 0 \\ 1 & -2 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$$A - I = \begin{bmatrix} -2 & 2 & 3 \\ 0 & 2 & -1 \\ 0 & 0 & 1 \end{bmatrix} \qquad \left(A - I\right)^{-1} = \frac{1}{-4} \begin{bmatrix} 2 & -2 & -8 \\ 0 & -2 & -2 \\ 0 & 0 & -4 \end{bmatrix} = \frac{1}{2} \begin{bmatrix} -1 & 1 & 4 \\ 0 & 1 & 1 \\ 0 & 0 & 2 \end{bmatrix}$$

$$X = \left(2B - C\right)\left(A - I\right)^{-1} = \begin{bmatrix} 3 & -1 & 0 \\ 1 & -2 & 0 \\ 0 & 0 & 0 \end{bmatrix} \frac{1}{2} \begin{bmatrix} -1 & 1 & 4 \\ 0 & 1 & 1 \\ 0 & 0 & 2 \end{bmatrix} = \frac{1}{2} \begin{bmatrix} -3 & 2 & 11 \\ -1 & -1 & 2 \\ 0 & 0 & 0 \end{bmatrix} =$$

$$= \begin{bmatrix} -1.5 & 1 & 5.5 \\ -0.5 & -0.5 & 1 \\ 0 & 0 & 0 \end{bmatrix}.$$

Task 2.14

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$$X=\begin{bmatrix}2&-3&1\\4&-5&2\\5&-7&3\end{bmatrix}^{-1}\begin{bmatrix}2&0&-2\\18&12&9\\23&15&11\end{bmatrix}\begin{bmatrix}9&7&6\\1&1&2\\1&1&1\end{bmatrix}^{-1}=$$

$$=\begin{bmatrix}-1&2&-1\\-2&1&0\\-3&-1&2\end{bmatrix}\begin{bmatrix}2&0&-2\\18&12&9\\23&15&11\end{bmatrix}\frac{1}{-2}\begin{bmatrix}-1&-1&8\\1&3&-12\\0&-2&2\end{bmatrix}=-\frac{1}{2}\begin{bmatrix}-2&-2&-2\\-2&-4&-6\\-4&-6&-2\end{bmatrix}=\begin{bmatrix}1&1&1\\1&2&3\\2&3&1\end{bmatrix}.$$

Task 2.15

$$AX+2B=C+BX$$
$$AX-BX=C-2B$$
$$(A-B)^{-1}\cdot/(A-B)X=C-2B$$
$$X=(A-B)^{-1}(C-2B);$$

$$A-B=\begin{bmatrix}2&0&6\\0&-2&2\\0&0&-1\end{bmatrix}\qquad (A-B)^{-1}=\frac{1}{4}\begin{bmatrix}2&0&12\\0&-2&-4\\0&0&-4\end{bmatrix}=\frac{1}{2}\begin{bmatrix}1&0&6\\0&-1&-2\\0&0&-2\end{bmatrix}$$

$$2B=\begin{bmatrix}-2&4&-6\\0&8&4\\0&0&4\end{bmatrix}\qquad C-2B=\begin{bmatrix}5&-4&7\\0&-6&0\\0&0&-3\end{bmatrix}$$

$$X=(A-B)^{-1}(C-2B)=\frac{1}{2}\begin{bmatrix}1&0&6\\0&-1&-2\\0&0&-2\end{bmatrix}\begin{bmatrix}5&-4&7\\0&-6&0\\0&0&-3\end{bmatrix}=\frac{1}{2}\begin{bmatrix}5&-4&-11\\0&6&6\\0&0&6\end{bmatrix}=$$

$$=\begin{bmatrix}2.5&-2&-5.5\\0&3&3\\0&0&3\end{bmatrix}.$$

Task 2.16

$$AXB^{-1}=I-A/\cdot B$$
$$A^{-1}\cdot/AX=(I-A)B$$
$$X=A^{-1}(I-A)B$$
$$X=(A^{-1}-I)B;$$

$$A^{-1}=\begin{bmatrix}1&11&-14\\0&-3&4\\-1&-12&15\end{bmatrix}\qquad A^{-1}-I=\begin{bmatrix}0&11&-14\\0&-4&4\\-1&-12&14\end{bmatrix}$$

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$$X=\left(A^{-1}-I\right)B=\begin{bmatrix}0 & 11 & -14\\ 0 & -4 & 4\\ -1 & -12 & 14\end{bmatrix}\begin{bmatrix}1 & 0 & 2\\ 1 & -2 & 0\\ 0 & -1 & 1\end{bmatrix}=\begin{bmatrix}11 & -8 & -14\\ -4 & 4 & 4\\ -13 & 10 & 12\end{bmatrix}.$$

Task 2.17

$$\left(A+3I\right)^{-1}\cdot /\left(A+3I\right)\left(X-I\right)=B$$
$$X-I=\left(A+3I\right)^{-1}B$$
$$X=I+\left(A+3I\right)^{-1}B;$$

$$A+3I=\begin{bmatrix}1 & 5 & -2\\ 2 & 11 & 0\\ -1 & -5 & 1\end{bmatrix}\qquad \left(A+3I\right)^{-1}=\begin{bmatrix}-11 & -5 & -22\\ 2 & 1 & 4\\ -1 & 0 & -1\end{bmatrix}$$

$$X=I+\left(A+3I\right)^{-1}B=\begin{bmatrix}1 & 0 & 0\\ 0 & 1 & 0\\ 0 & 0 & 1\end{bmatrix}+\begin{bmatrix}-11 & -5 & -22\\ 2 & 1 & 4\\ -1 & 0 & -1\end{bmatrix}\begin{bmatrix}-3 & 21 & 1\\ 2 & 50 & -2\\ 1 & -22 & 0\end{bmatrix}=\begin{bmatrix}2 & 3 & -1\\ 0 & 5 & 0\\ 2 & 1 & 0\end{bmatrix}.$$

Task 2.18 a) $R\left(A\right)=2,$ b) $R\left(B\right)=3,$ c) $R\left(C\right)=2,$ d) $R\left(D\right)=6.$

Task 2.19 $X=\frac{1}{12}\begin{bmatrix}10\\ 15\\ -7\end{bmatrix}.$

Task 2.20 Yes, it can. $X=\begin{bmatrix}0.5\\ -2\end{bmatrix}.$

Task 2.21 $X=\begin{bmatrix}1/7\\ u_1+u_2\\ u_1+u_2-3/7\\ u_1-6/7\\ u_2\end{bmatrix},\quad u_1,u_2\in\mathbb{R}$. The vector X^* is not the solution because it does not meet the 3rd and 4th equation of the system.

Task 2.22 The system has no solution (therefore, it is an impossible system) because

$$r\left(A\right)=3\neq 4=r\left(\tilde{A}\right).$$

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Task 2.23 $X = \begin{bmatrix} \frac{2}{3} - u_3 - u_2 \\ u_2 - u_1 + 3u_3 \\ u_1 \\ u_2 \\ u_3 + \frac{1}{3} \end{bmatrix}, \quad u_1, u_2, u_3 \in \mathbb{R}$.

Task 2.24 The system is impossible for $p = -2$. There is no $p \in \mathbb{R}$ for which the system has infinite number of solutions.. Namely, for $p \neq -2$ is $m = n = r(A) = r(\tilde{A}) = 3$ the system has a unique solution.

Vector $X = \begin{bmatrix} 1/3 \\ -1/3 \\ -2 \end{bmatrix}$ is a solution to the system for $p = 1$.

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2.14. MATRIX - QUIZ

Circle the answer that you think the best complements the statement.

1 If the matrix A is $A=\begin{bmatrix} 2 & 3 & 0 \\ -1 & 4 & -2 \end{bmatrix}$ and $B=\begin{bmatrix} 4 & -3 & 5 \\ -2 & 1 & 6 \end{bmatrix}$, then is $6A+7B=$

○ $\begin{bmatrix} 40 & 3 & 35 \\ -20 & 31 & 30 \end{bmatrix}$ ○ $\begin{bmatrix} -40 & -3 & 35 \\ -20 & 31 & 30 \end{bmatrix}$ ○ $\begin{bmatrix} 40 & -3 & 35 \\ -20 & 31 & 30 \end{bmatrix}$ ○ $\begin{bmatrix} 40 & -3 & 35 \\ 20 & 31 & 30 \end{bmatrix}$

2. $\begin{vmatrix} 3 & 4 & 5 \\ -1 & 0 & 7 \\ 2 & 4 & -3 \end{vmatrix} =$

◉ 60 ○ - 60 ○ -4 ○ 4

3. A matrix has 72 elements. The number of rows in this matrix cannot be:

○ 4 ○ 5 ○ 6 ○ 7

4. Use the Cramer's rule to solve the system

$$\begin{aligned} 0.5x + 1.4y &= 9 \\ 0.3x - 0.2y &= 0.2 \end{aligned}$$

The solution of this system is a vector:

○ $\begin{bmatrix} 5 \\ 4 \end{bmatrix}$ ○ $\begin{bmatrix} -4 \\ -5 \end{bmatrix}$ ○ there is no solution ○ $\begin{bmatrix} 4 \\ 5 \end{bmatrix}$

5. If the matrix A is $A=\begin{bmatrix} 6 & 3 & 9 & 9 & -1 \\ 8 & -8 & 9 & 0 & 8 \\ -7 & -4 & -7 & 6 & 6 \\ 8 & 1 & 9 & -7 & 9 \end{bmatrix}$, then is $a_{32}=$

○ 9 ○ -7 ○ -4 ○ 6

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