$$\det A=\begin{vmatrix}5&4&5&4\\1&2&1&4\\2&3&5&4\\0&2&1&1\end{vmatrix}\begin{matrix}R_1-5R_2\\ \\R_3-2R_2\\ \\ \end{matrix}\overset{6)}{=}\begin{vmatrix}0&-6&0&-16\\1&2&1&4\\0&-1&3&-4\\0&2&1&1\end{vmatrix}=\sum_{k=1}^{4}a_{k1}A_{k1}=a_{21}A_{21}=$$

$$=1\cdot(-1)^{2+1}\begin{vmatrix}-6&0&-16\\-1&3&-4\\2&1&1\end{vmatrix}R_2-3R_3\overset{6)}{=}-\begin{vmatrix}-6&0&-16\\-7&0&-7\\2&1&1\end{vmatrix}=-\sum_{k=1}^{3}a_{k2}A_{k2}=-a_{32}A_{32}=$$

$$=-1\cdot(-1)^{3+2}\begin{vmatrix}-6&-16\\-7&-7\end{vmatrix}=\begin{vmatrix}-6&-16\\-7&-7\end{vmatrix}\overset{8)}{=}-7\begin{vmatrix}-6&-16\\1&1\end{vmatrix}\overset{8)}{=}14\begin{vmatrix}3&8\\1&1\end{vmatrix}=14(3-8)=-70.$$

Method 2 (Laplace expansion by the 4th row):

$$\det A=\begin{vmatrix}5&4&5&4\\1&2&1&4\\2&3&5&4\\0&2&1&1\end{vmatrix}\overset{6)}{=}\begin{vmatrix}5&-6&5&-1\\1&0&1&3\\2&-7&5&-1\\0&0&1&0\end{vmatrix}=\sum_{k=1}^{4}a_{4k}A_{4k}=a_{43}A_{43}=1\cdot(-1)^{4+3}\begin{vmatrix}5&-6&-1\\1&0&3\\2&-7&-1\end{vmatrix}\overset{6)}{=}$$

$$\overset{6)}{=}-\begin{vmatrix}5&-6&-16\\1&0&0\\2&-7&-7\end{vmatrix}=-\sum_{k=1}^{3}a_{2k}A_{2k}=-a_{21}A_{21}=-1\cdot(-1)^{2+1}\begin{vmatrix}-6&-16\\-7&-7\end{vmatrix}=\begin{vmatrix}-6&-16\\-7&-7\end{vmatrix}=-70.$$

2.7. INVERSE MATRIX

Let A be a real square matrix of order n.

The matrix A is a *regular* if $\det A \neq 0$. The matrix A is *singular* if $\det A = 0$.

Only regular matrix has an inverse matrix.

For a regular matrix A of order n there is a unique matrix B such that

$$A \cdot B = B \cdot A = I_n,$$

where I_n is a unit matrix of order n. The matrix B is also a regular matrix of order n, marked by A^{-1}, and is called *the inverse matrix* of the matrix A.

2.8. DETERMINING THE INVERSE MATRIX BY CALCULATING DETERMINANTS

The inverse matrix $\boldsymbol{A}^{-1}$ of a regular matrix $\boldsymbol{A}$ of order $\boldsymbol{n}$ can be determined by the formula:

$$\boldsymbol{A}^{-1}=\frac{1}{\det A}\begin{bmatrix}A_{11}&A_{21}&\cdots&A_{n1}\\A_{12}&A_{22}&\cdots&A_{n2}\\\vdots&\vdots&\ddots&\vdots\\A_{1n}&A_{2n}&\cdots&A_{nn}\end{bmatrix},$$

where $\boldsymbol{A}_{ij}$ is the cofactor of the element $\boldsymbol{a}_{ij}$ of the matrix $\boldsymbol{A}$.

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