9.12 CONNECTIONS AND APPLICATIONS

9.12.1 Reliability - application of Poisson distribution.

The probability of k failures in a unit of time is given by

$$f(k) = e^{-\lambda}\frac{\lambda^k}{k!},$$

where λ is average number of failures (expected number of failures) in a unit of time.

A function

$$f(k;\lambda,t) = e^{-\lambda t}\frac{(\lambda t)^k}{k!}$$

is a probability of k failures in time t.

Note that a probability of 0 failures in time t is given by

$$f(0;\lambda,t) = e^{-\lambda t}\frac{(\lambda t)^0}{0!} = e^{-\lambda t}.$$

Definition: Reliability function

Reliability function $R(t)$ is a probability of zero failures at time interval of length t. It is given by

$$R(t) = \begin{cases} 1, & t < 0 \\ e^{-\lambda t}, & t \geq 0 \end{cases}.$$

Probability of no more than m failures at time interval of length $t > 0$ is given by

$$R(t,m) = \sum_{k=0}^{m} e^{-\lambda t}\frac{(\lambda t)^k}{k!}.$$

Exercise 9.9

Some complex system has an average failure rate $\lambda = 0.002$ transistor failures per hour. What is the reliability for a 30 days period if the number of transistor failures cannot exceed 1?

Solution:

$\lambda = 0.002$
$t = 30 \cdot 24 = 720$
$m \leq 1$
$\lambda t = 1.44$

We have to compute $R(720,1)$.

$R(720,1) = e^{-1.44} + e^{-1.44} \cdot 1.44 \approx 0.58$

Co-funded by the
Erasmus+ Programme
of the European Union