9.13 EXERCISES

Exercise 9.10

1. How many two-digit odd natural numbers greater than 30 are there?
2. How many three-digit even natural numbers less than 500 are there?
3. In how many ways can we arrange the letters of the word $MONTANA$?
4. There are seven harbours in a country A. In how many ways can four different ships dock there?
5. In how many ways can six coins be hidden in four boxes.
6. Six points lie on the circumference of a circle. How many of inscribed triangles can be drawn having these points as vertices?
7. If the letters of word *algebra* are placed at random in a row, what is the probability that two successive letters will be a.
8. If the letters of word *about* are placed at random in a row, what is the probability that three successive letters will be vowels.
9. A couple wants to have three children. We suppose that the gender of the child is equally likely. Give the probabilities that:
   a) the couple has at least one boy,
   b) there is no girl older than a boy,
   c) the couple has exactly one girl.
10. Two ships A and B have arriving time between $\mathbf{1}$ pm and $\mathbf{5}$ pm. Both ships must dock on the same berth. Once docked, it takes each ship $\mathbf{30}$ minutes to restock and leave the dock. What is the probability
    a) that the ships won't have to wait for the berth?
    b) that the ship A won't have to wait for the berth?
    c) that the ship B will have to wait for the berth?
11. Suppose that two balls are drawn without replacement (the first ball is not replaced before the second is drawn) at random from a bag containing 4 red and 3 black balls. Let A be the event of drawing a red ball the first time, and B the event of drawing a red ball the second time. Are the events A and B independent?
12. Three identical bowls are labelled $1, 2, 3$. First bowl contains 3 red and 3 blue marbles. Second bowl contains 4 red and 2 blue marbles. Third bowl contains 1 red and 5 blue marbles. A bowl is randomly selected, and a marble is randomly selected from the

Co-funded by the Erasmus+ Programme of the European Union

bowl. a) What is the provability that a marble selected is blue? b) Given that a marble selected is red, what is the probability that bowl 2 was selected?

13. Suppose that the probability that John will solve a certain problem is $\frac{2}{3}$, that Mary will solve it is $\frac{3}{4}$ and that Bill will solve it is $\frac{1}{2}$. What is the probability
    d) that at least one person will solve it?
    e) that Mary and Bill will solve it but John will not?
    f) that John and Mary will solve it but Bill will not?
    g) that at least two people will solve it?

14. A box contains three coins: two regular coins and one fake two-headed coin $(P(H) = 1)$. You pick a coin at random and toss it.
    a) What is the probability that it lands heads up?
    b) You pick a coin at random and toss it and get heads. What is the probability that it is the two-headed coin?

15. You toss a fair coin three times:
    a) What is the probability of three heads?
    b) What is the probability that you observe exactly one heads?
    c) Given that you have observed at least one heads, what is the probability that you observe at least two heads?

16. Articles coming through an inspection line are visually inspected by two successive inspectors. When a defective article comes through the inspection line, the probability that it gets by the first inspector is 0.1. The second inspector will miss five out of ten of the defective items that get past the first inspector. What is the probability that a defective item gets by both inspectors?

17. In an exam, two reasoning problems, 1 and 2, are asked. 35% students solved problem 1 and 15% students solved both the problems. How many students who solved the first problem will also solve the second one?

18. Out of 50 people in a group, 35 smoke in which there are 20 males and 15 do not smoke in which there are 10 females. What is the probability that if the person taken at random is a male then he is a smoker?

19. A certain disease has an incidence rate of 2%. Suppose that for some diagnostic test the false negative rate is 1% and false positive rate is 1%. Compute the probability that a person, chosen at random from the population:
    a) who tests positive actually has the disease.
    b) who tests negative actually has not the disease

20. The probability distribution for a random variable X is given in table.

x_i	-3	-1	0	2	3
p_i	0.1	0.4	0.2	0.2	0.1

Find the mean, variance, and standard deviation of X.

21. Five out of hundred men and two out of 1000 women are color-blind persons. From a group of the same number of men and women one person was taken at random. This person was found to be a colour-blind. What is the probability that it was a male?

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22. Three identical bowls are labelled $1, 2, 3$. First bowl contains 3 red and 4 blue and 3 black marbles. Second bowl contains 6 red and 2 blue and 2 black marbles. Third bowl contains 2 red and 5 blue and 3 black marbles. A bowl is randomly selected, and a marble is randomly selected from the bowl.

    a) What is the provability that a marble selected is black?
    b) Given that a marble selected is black, what is the probability that bowl 3 was selected?
    c) Given that a marble selected is blue, what is the probability that bowl 1 was selected?

23. Three identical bowls are labelled $1, 2, 3$. First bowl contains 3 red and 4 blue and 3 black marbles. Second bowl contains 6 red and 2 blue and 2 black marbles. Third bowl contains 2 red and 5 blue and 3 black marbles. A bowl is randomly selected, and two marbles are randomly selected without replacement from the bowl.

    a) What is the provability that both marbles selected are blue?

    b) Given that both marbles selected are blue, what is the probability that bowl 2 was selected?

    c) Given that a marble selected are blue and black, what is the probability that bowl 2 was selected?

24. A supervisor in a factory has three men and three women working for him. He wants to choose two workers for a special job. He decides to select the two workers at random. Let Y denote the number of women in his selection. Find the probability distribution for Y.

25. Each of three balls are randomly placed into one of three bowls. Find the probability distribution for Y = the number of empty bowls.

26. A balanced coin is tossed three times. Let Y equal the number of heads observed.
    a) Calculate the probabilities associated with $Y = 0, 1, 2$, and 3.

b) Construct a probability distribution table.

c) Find the expected value and standard deviation of Y.

27. An insurance company issues a one-year $\$2000$ policy insuring against an occurrence A that historically happens to 1 out of every 100 owners of the policy. How much should the company charge for the policy if it requires that the expected profit per policy be $\$75$?

28. A basketball player takes 4 independent freethrows with a probability of 0.7 of getting a basket on each shot. Let Y = the number of baskets he gets. Find the probability distribution for a random variable Y. Find the probability that he gets at least 3 baskets.

29. Suppose that a radio contains six transistors, two of which are defective. Three transistors are selected at random, removed from the radio, and inspected. Let X equal the number of defectives observed. Find the probability distribution for X.

30. Suppose that two balls are drawn with replacement (the first ball is replaced before the second is drawn) at random from a bag containing 5 red and 3 black balls. Let X is equal to the number of black balls drown. Find the probability distribution for a random variable X .

31. Suppose that two balls are drawn with no replacement at random from a bag containing 5 red and 3 black balls. Let X is equal to the number of black balls drown. Find the probability distribution for a random variable X .

32. The probability distribution for a random variable X is given in table.

x_i	0	2	4	5	6
p_i	$\frac{1}{3}$	$\frac{1}{12}$	$\frac{1}{3}$	$\frac{1}{6}$	$\frac{1}{12}$

a) Find the cumulative distribution function of X. Sketch the graph.

b) Calculate: i) $P(2 \leq X \leq 4)$ ii) $P(0 < X < 4)$ iii) $P(X > 1)$.

33. A football player takes three independent penalties with a probability $\frac{4}{5}$ of scoring a goal on each shot. Let X be the number of goals he scores. Find the probability distribution for a random variable X. Find the expected number of goals.

34. A variable X has a normal distribution with a mean of 10 and a standard deviation of 2. One score is randomly sampled. What is the probability that it is between 11 and 12?

35. Sea depth was measured in 5 independent trials, and the results were (in meters): $862, 870, 876, 866, 871$. Knowing that the distribution of measurements is normal with standard deviation of $5\,m$, and the significance level being $\alpha = 0{,}05$, verify the hypothesis that the average sea depth in that area is $870m$.

36. In a biochemical experiment, lifespan of certain organisms was measured. Distribution of that time can be assumed as normal. 8 measurements were taken, and the results were (in hours): $4.7;\ 5.3;\ 4.0;\ 3.8;\ 6.2;\ 5.5;\ 4.5;\ 6.0$. Assuming the significance level $\alpha = 0{,}05$, assess the hypothesis that the average lifespan of these organisms is 4.0 hours.

37. Find a linear regression function for data: $x = -1, 1, 2, \quad y = 1, -2, 2.$

38. Some complex system has an average failure rate $\lambda = 0.005$ lamp failures per hour. What is the reliability for a 60 days period if the number of lamp failures cannot exceed 2?

Solutions:

1. 35
2. 200
3. 1260
4. 720
5. 4096
6. 20
7. $\frac{2}{7}$
8. $\frac{1}{10}$
9. a) $\frac{7}{8}$ b) $\frac{1}{2}$ c) $\frac{3}{8}$
10. a) ≈ 0.7656 b) ≈ 0.8828 c) ≈ 0.1172
11. not
12. a) $\frac{5}{9}$ b) $\frac{1}{2}$
13. a) $\frac{23}{24}$ b) $\frac{1}{8}$ c) $\frac{1}{4}$ d) $\frac{17}{24}$
14. a) $\frac{2}{3}$ b) $\frac{1}{2}$
15. a) $\frac{1}{8}$ b) $\frac{3}{8}$ c) $\frac{4}{7}$
16. 0.05
17. 42.8%

18. $\frac{4}{5}$
19. a) ≈ 0.6689 b) ≈ 0.9998
20. $0,\ 3,\ \sqrt{3}$
21. $\frac{25}{26}$
22. a) $\frac{4}{15}$ b) $\frac{3}{8}$ c) $\frac{4}{11}$
23. a) $\frac{17}{135}$ b) $\frac{1}{17}$ c) $\frac{4}{393}$
24. $P(Y=0)=\frac{1}{5},\ P(Y=1)=\frac{3}{5},\ P(Y=2)=\frac{1}{5}.$
25. $P(Y=0)=\frac{2}{9},\ P(Y=1)=\frac{2}{3},\ P(Y=2)=\frac{1}{9}.$
26. a) $P(Y=0)=\frac{1}{8},\ P(Y=1)=\frac{3}{8},\ P(Y=2)=\frac{3}{8},\ P(Y=3)=\frac{1}{8}$ c) $E(Y)=1.5,$ $V(Y)=0.75.$
27. \$95
28. $P(Y=0)=0.0081, P(Y=1)=0.0756, P(Y=2)=0.2646, P(Y=3)=0.4116,$ $P(Y=4)=0.2401,\ P(Y\geq 3)=0.6517.$
29. $P(X=0)=\frac{1}{5},\ P(X=1)=\frac{3}{5}, P(X=2)=\frac{1}{5}.$
30. $P(X=0)=\frac{25}{64},\ P(X=1)=\frac{15}{32},\ P(X=2)=\frac{9}{64}.$
31. $P(X=0)=\frac{5}{14},\ P(X=1)=\frac{15}{28},\ P(X=2)=\frac{3}{28}.$
32. a) $F(x)=0\ x<0,\ F(x)=\frac{1}{3}\ 0\leq x<2,\ F(x)=\frac{5}{12}\ 2\leq x<4,\ F(x)=\frac{3}{4}\ 4\leq x<5,\ F(x)=\frac{11}{12}\ 5\leq x<6,\ F(x)=1\ x\geq 6.$ b) i) $\frac{5}{12}$ ii) $\frac{1}{12}$ iii) $\frac{2}{3}.$
33. $P(X=0)=\frac{1}{125},\ P(X=1)=\frac{12}{125},\ P(X=2)=\frac{48}{125},\ P(X=3)=\frac{64}{125}.$ $E(X)=2.4.$
34. 0.1499
35. $|u|=0.447<1.96=u_\alpha,\ H_0$ cannot be rejected
36. $|t|=3.17>2.365=t_\alpha,\ H_0$ must be rejected
37. $y\ \cong 0.0714x+0.2857$
38. ≈ 0.0255