9.2 EVENTS. PROBABILITY

Example 9.12

Suppose that we conduct an experiment by tossing two different fair coins. Let H represents heads and T represents tails. Then any possible outcome of the experiment is an element of the set

$$\Omega = \{(H,H),(H,T),(T,H),(T,T)\}.$$

Definition: : Sample space, elementary event

A *sample space* is a set Ω of elements that correspond one to one with the outcomes of an experiment. Each of the elements of Ω is called an *elementary event*.

Definition: Event

An *event* is any subset of a sample space.

Remark

The sample space Ω is an event (*a certain event*, *sure event*), any elementary event is an event, the empty set $\emptyset$ is an event (*an impossible event*).

Example 9.13

Each letter of the word *leopard* is written on a separate card and the cards are shuffled. List a sample space for the outcome of drawing one card.

Answer: $\Omega = \{l,e,o,p,a,r,d\}$ (also $\Omega = \{e,p,l,a,d,o,r\}$ since the order is not important).

Now suppose we are interested in whether the letter drawn is a vowel. We call the drawing of a vowel an event A. The event A is the occurrence of any of the outcomes e, o, a. It can be seen as the set $A = \{e,o,a\}$ which is a subset of the sample space Ω.

Definition: Classical definition of probability

Let Ω be a sample space of an experiment in which there are n possible outcomes, each equally likely. If an event A is a subset of Ω such that A contains k elements, then the probability of an event A, denoted by $P(A)$, is given by

$$P(A) = \frac{k}{n}.$$

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Example 9.14

Suppose that we conduct an experiment by tossing two different six-sided dices. What is the probability that we get ten in total?

Solution:

The number of elementary events is $n = 6 \cdot 6 = 36.$ Denote by A the event that we get the sum ten. We have

$$A = \{(6,4),(5,5),(4,6)\}$$

hence $k = 3$ and

$$P(A) = \frac{3}{36} = \frac{1}{12}.$$

Let Ω be now a finite or infinite set. We will call Ω a sample space. Denote by F the class of subsets of $\Omega.$ Elements of F will be called events. If Ω is finite, all its subsets are events. If Ω is not finite, we consider as events a large infinite class of Ω subsets.

Definition: Probability (axiomatic)

A probability of an event A is a real number $P(A)$ which satisfies the following three conditions:

1. $P(A) \geq 0$

2. $P(\Omega) = 1$

3. For every sequence $A_1, A_2, \ldots$ of mutually exclusive events it holds

$$P(A_1 \cup A_2 \cup \ldots) = P(A_1) + P(A_2) + \cdots.$$

Properties of probability

1. $P(\emptyset) = 0$
2. $0 \leq P(A) \leq 1$
3. $P(A \cup B) = P(A) + P(B) - P(A \cap B)$
4. $A \subset B \Rightarrow P(A) \leq P(B)$
5. $P(\bar{A}) = 1 - P(A)$
6. $P(A \setminus B) = P(A) - P(A \cap B)$
7. $B \subset A \Rightarrow P(A \setminus B) = P(A) - P(B)$
8. $$P(A \cup B \cup C) = P(A) + P(B) + P(C) - P(A \cap B) - P(A \cap C) - P(B \cap C) + P(A \cap B \cap C)$$

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