9.3 GEOMETRIC PROBABILITY

As a sample space we consider a 1D, 2D or 3D set with a finite measure $m(\Omega)$ length, area or volume respectively. As events we consider subsets of Ω with finite measure.

Definition: geometric probability

Probability that a point x lying in a set Ω lies also in its subset A with a finite measure $m(A)$ is defined as the ratio

$$P(A) = \frac{m(A)}{m(\Omega)}.$$

Example 9.15

A circle is inscribed in a square. A point is selected at random from the area of the square. Calculate the probability that it lies inside the circle.

Solution:

Let Ω be a square and A be a circle of radius $r > 0$ that is inscribed in Ω. We have

$$m(A) = \pi r^2, \qquad m(\Omega) = 4r^2.$$

$$P(A) = \frac{\pi r^2}{4r^2} = \frac{\pi}{4}$$

Example 9.16

Two ships have arriving time between $\mathbf{2}$ pm and $\mathbf{3}$ pm. Both ships must dock on the same berth. Once docked, it takes each ship $\mathbf{10}$ minutes to restock and leave the dock. What is the probability that the ships won't have to wait for the berth?

Solution:

As a unit of time, we set 10 minute. A time interval between 2 and 3 pm we denote by $[0,6]$. Let x be the time when the first boat will arrive, and y be the time when the second boat will arrive. We have $x, y \in [0,6]$ and $|x - y| \geq 1$.

Let $\Omega = [0,6] \times [0,6]$ be a sample space. The area of Ω is $\mu(\Omega) = 36$.

The event that both ships won't have to wait can be represented by the set

$$A = \{(x, y) \in \Omega : |x - y| \geq 1\}.$$

Since

$$A = \{(x, y) \in \Omega : y \leq x - 1 \text{ or } y \geq x + 1\},$$

the area of the set A is

$$\mu(A) = \frac{1}{2} \cdot 5 \cdot 5 + \frac{1}{2} \cdot 5 \cdot 5 = 25.$$

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From the geometric probability formula, we have

$$P(A) = \frac{m(A)}{m(\Omega)} = \frac{25}{36} \approx 0.6944.$$

There is a 69.44% chance that the ships won't have to wait for the berth.

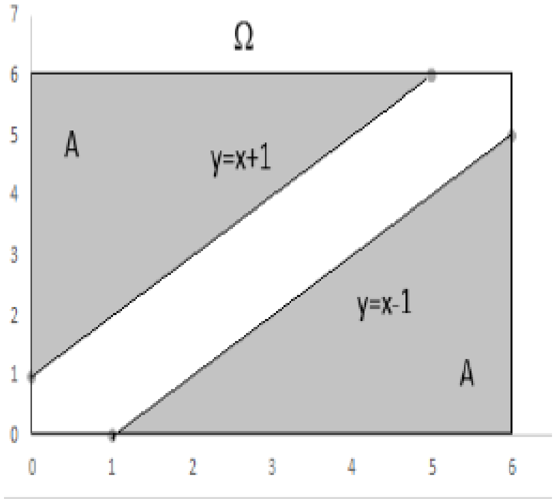

Figure 9.1. Illustration for Example 9.16

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