9.8 NORMAL DISTRIBUTION

Definition: Normal distribution

A *normal (Gaussian) distribution* is a continuous distribution with density

$$f(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{(x-\mu)^2}{2\sigma^2}}, \qquad -\infty < x < \infty.$$

It is denoted by $N(\mu, \sigma)$, where μ is mean and σ is standard deviation.

Definition: Standard normal distribution

A *standard normal (Gaussian) distribution* is a normal (Gaussian) distribution with mean $\mu = 0$ and variance $\sigma^2 = 1$. Probability density function is given by

$$f(x) = \frac{1}{\sqrt{2\pi}} e^{-\frac{x^2}{2}}, \qquad -\infty < x < \infty.$$

It is denoted by $N(0,1)$.

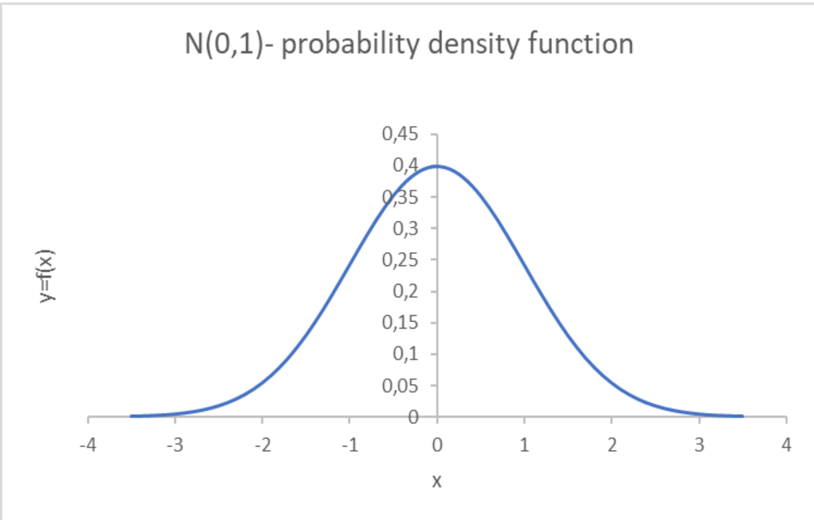

Figure 9.4: Standard normal distribution

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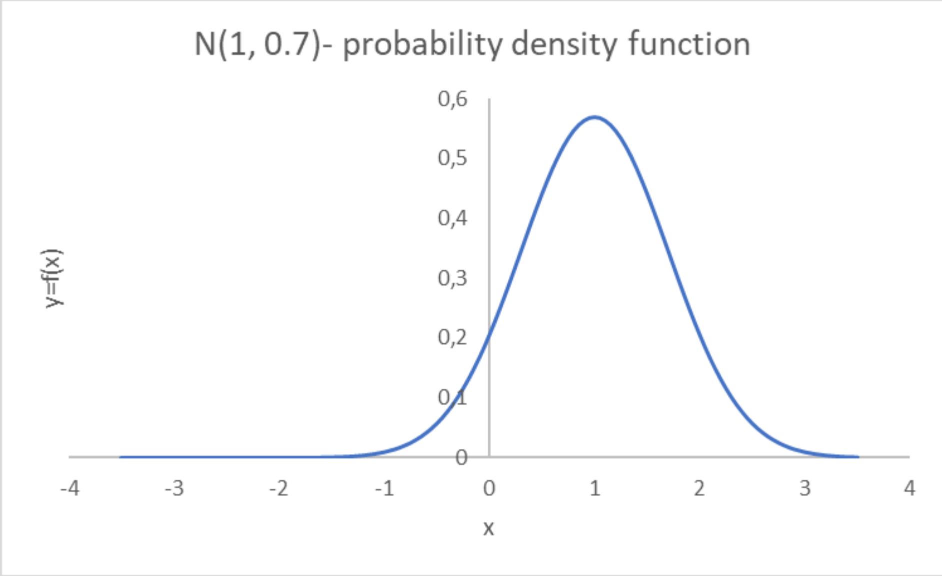

Figure 9.5: *Normal distribution* $\mu = 1, \sigma = 0.7$

Remark:

The graph of normal distribution density function is called Gaussian curve.

The *cumulative distribution function* of a normal random variable is given by

$$F(x) = \frac{1}{\sigma\sqrt{2\pi}} \int_{-\infty}^{\infty} e^{-\frac{(t-\mu)^2}{2\sigma^2}} dt, \qquad -\infty < x < \infty.$$

This integral is not elementary.

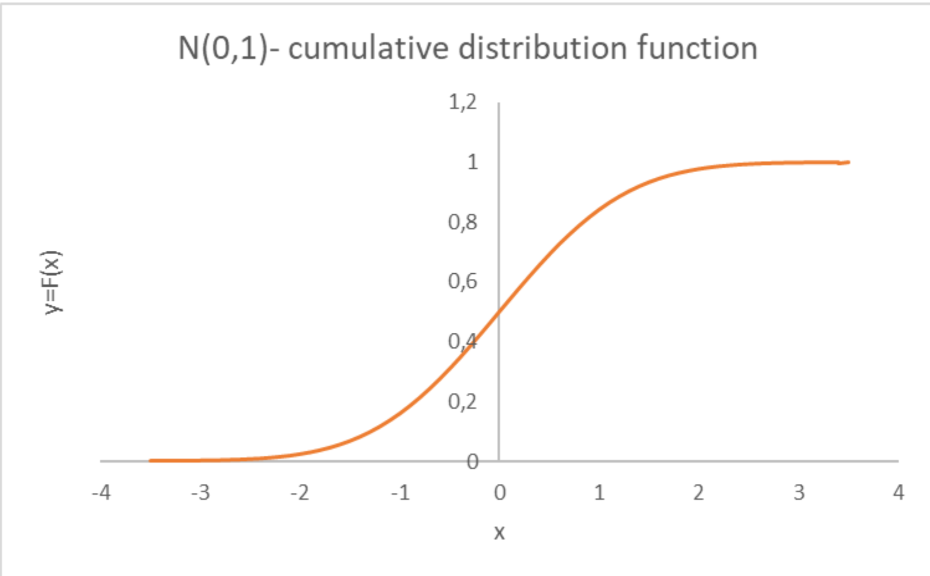

Figure 9.6. *Standard normal cumulative distribution function*

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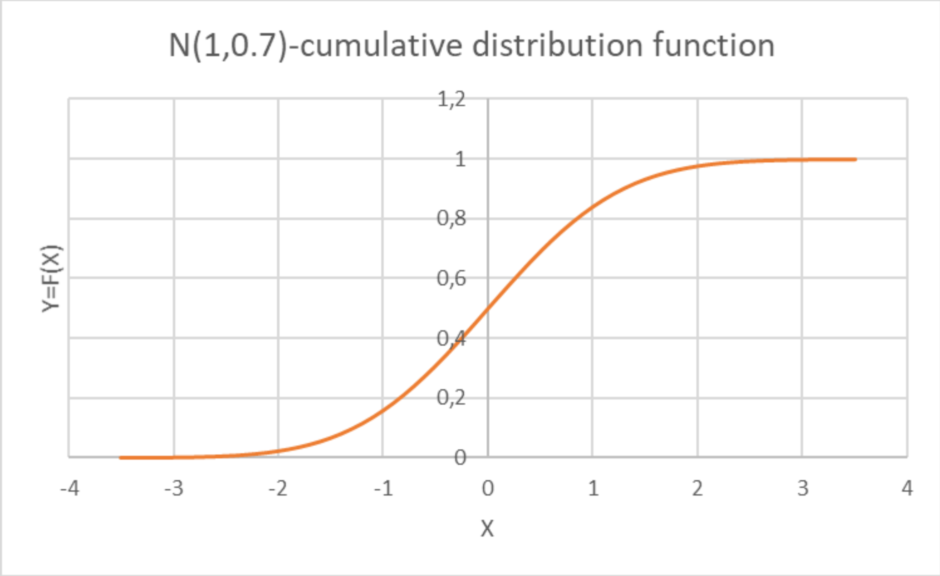

Figure 9.7. *Normal cumulative distribution function $\mu = 1, \sigma = 0.7$*

Example 9.29

A variable X has a normal distribution with a mean of 20 and a standard deviation of $3.$ One score is randomly sampled.

1. What is the probability that it is above 15?

2. What is the probability that it is in the interval $[18,22]$?

Solution:

1. A variable X has a distribution $N(20,3)$. We must compute $P(X > 15)$. Let F be the cumulative distribution function of X.

$$P(X > 15) = 1 - P(X \leq 15) = 1 - F(15) \approx 1 - 0.0478 = 0.9522$$

We find $F(15)$ by calculating NORM.DIST(15;20;3;1) in Excel or NORMDIST(15;20;3;1) in OpenOffice Calc.

2. $P(18 \leq X \leq 22) = F(22) - F(18) \approx 0.7475 - 0.2525 = 0.4950$

Exercise 9.7

Suppose that herring lengths is normally distributed with the mean of 8 inches and the standard deviation of 1.5 inches. A fishing vessel can catch 10 tons of herrings daily. Estimate:

1. How many of the fish are longer than 11 inches?
2. How many would you expect to be shorter than 6 inches?
3. How many have a length between 8 and 10 inches?

Solution:

1. $\approx 0.0228 \cdot 10\ t = 0.228\ t$
2. $\approx 0.0912 \cdot 10\ t = 0.912\ t$
3. $\approx 0.4088 \cdot 10\ t = 4.088\ t$

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