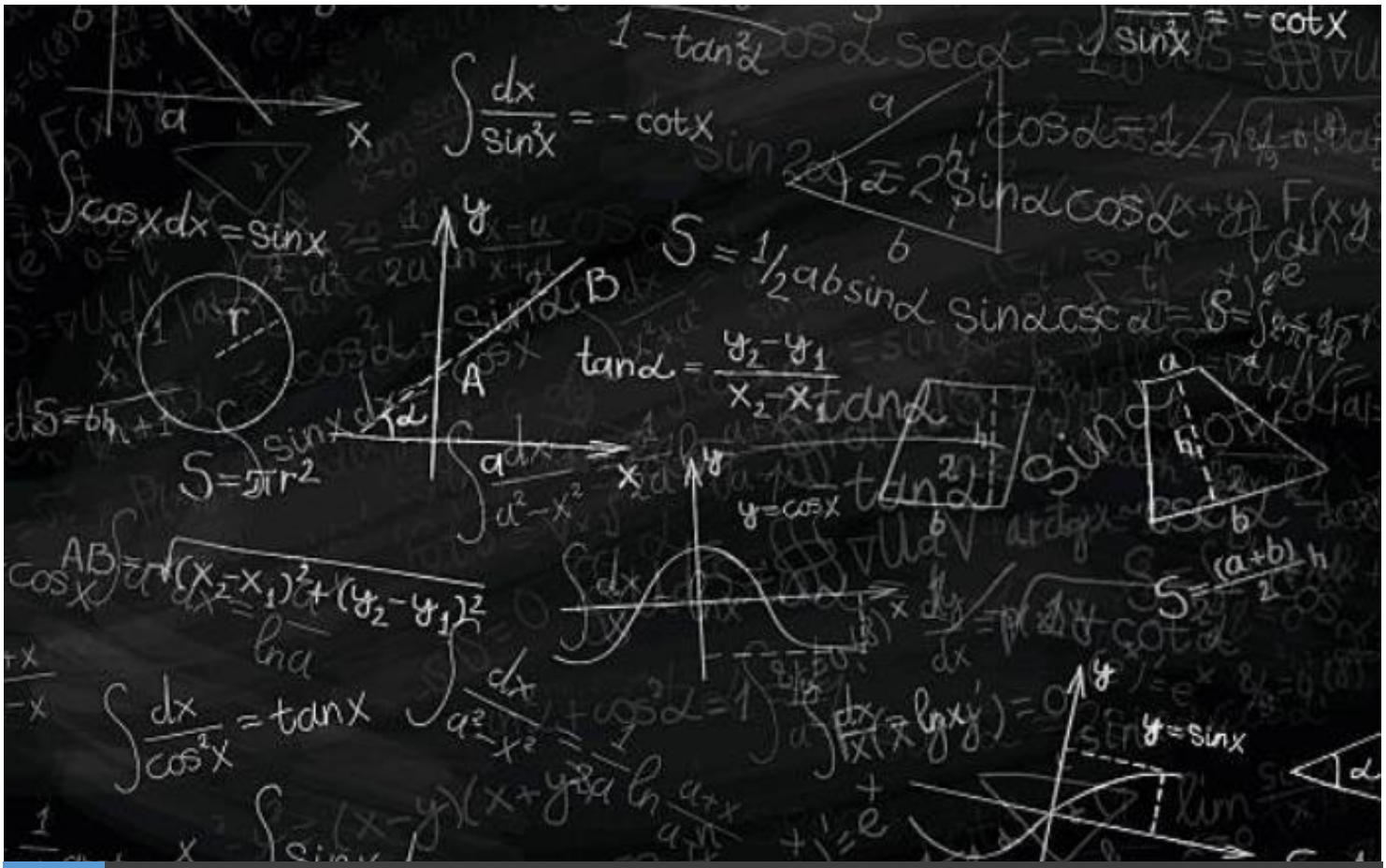
MARE Math/CS

Innovative Approach in Mathematical Education for Maritime Students



Teacher's Manual

Calculus

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MareMathics

Innovative Approach in Mathematical Education for Maritime Students

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https://maremathics.pfst.hr/

Manual for teachers

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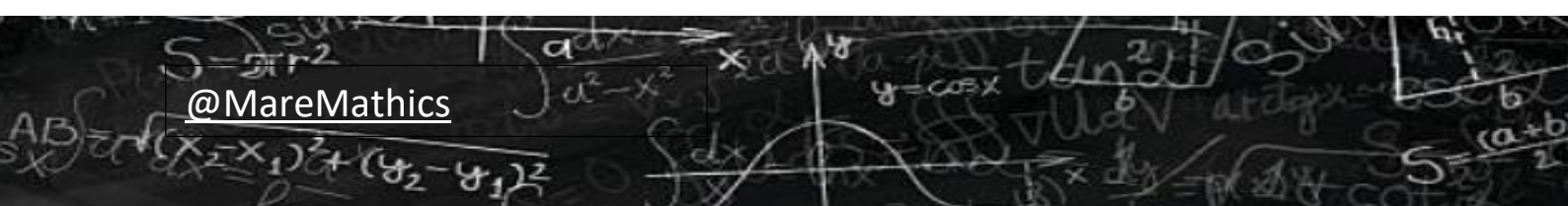

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The Manual is the outcome of the collaborative work of all the Partners for the development of the MareMathics Project.

Partners in the project:

University of Split Faculty of Maritime Studies **CROATIA**

Tallinn University of Technology, Estonian Maritime Academy **ESTONIA**

Latvian Maritime Academy **LATVIA**

Polish Naval Academy Gdynia **POLAND**

Toni Milun Independent Microenterprise **CROATIA**

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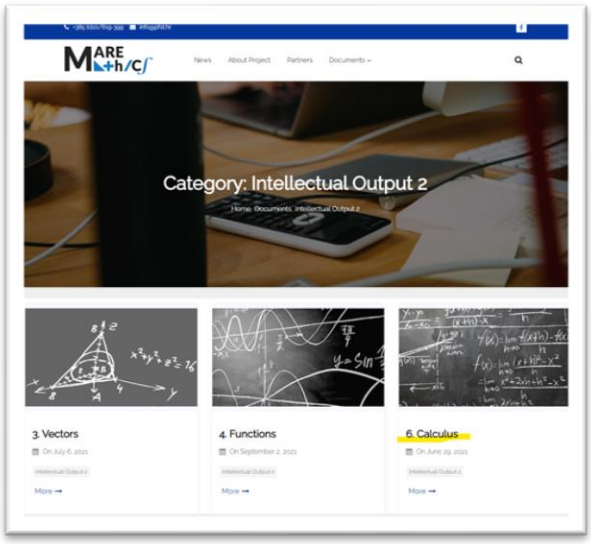
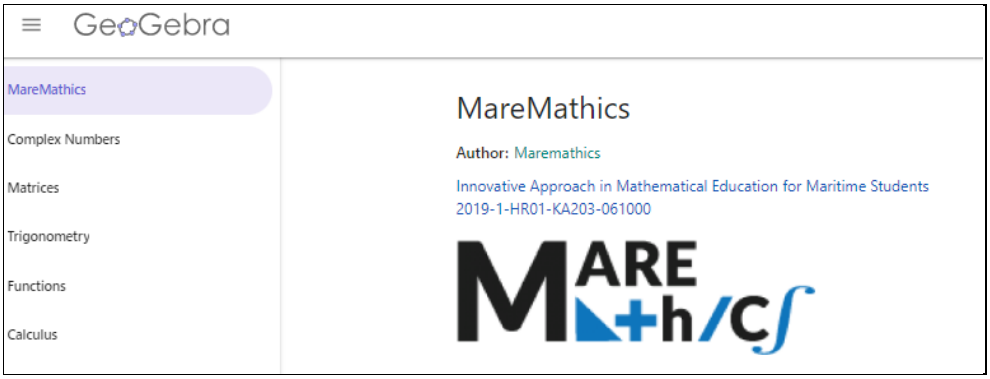
CONTENTS

CALCULUS: Teaching and Learning Plan

The goal of this material and related resources is to assist teachers in planning their lessons allowing achieving learning outcomes posted in the course's syllabus. It enables teachers to design student activities to encourage students to learn.

The resources are picked from project ***MareMathics*** and available on the https://maremathics.pfst.hr/.





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Name of Unit	Workload	Handbook
Calculus	Lecture: 380 min Exercises: 585 min	Unit 6. Calculus

DETAILED DESCRIPTION

Most first-year students find it hard to understand and acquire mathematical notions of differential calculus. This is often due to insufficient prior knowledge or because these notions are really difficult and require mathematical and logical maturity. Given the difficulties, this unit explains the matter gradually, starting with the targeted theoretical notions, which is followed by exercises and solved problems, with the aim of teaching the students how to solve tasks independently and how to apply the acquired knowledge in solving problem tasks in the area of maritime affairs.

Basic notions associated with the derivation of function are explained, along with the rules and techniques of derivatives. Particular attention is paid to the application of derivation in the problems of the tangent, the normal, the differential, and the establishing the function limits. The application of derivations in the flow examination and function graph drawing are explained and followed by the application of derivations in maritime affairs.

AIM: Acquire knowledge and skills in those areas of differential calculus which are necessary to follow the curricula of other courses of the study programme, and are expected to be implemented in maritime practice.

Learning Outcomes:

1. Define the notions of derivative, function limit and differential.
2. Apply simple and complex derivation rules when solving tasks.
3. Perform the derivation of the complex, parametrically or implicitly given function.
4. Explain the concept of the real variable of real functions and the geometric interpretation of the derivative at a point.
5. Apply the derivative in finding the local and global extremes of the function of a given variable, and the points of the function inflexion.
6. Analyse the flow of an elementary function by using derivation, and sketch its graph.

Prior Knowledge: sets and functions, sequences and series, limits and continuity of the function

Key words of this Unit: derivatives, applications, function limits, tangent and normal lines, graphs

Relationship to real maritime problems: mechanics (problem of speed), meteorology (weather forecast – extreme sea states), electronics (graphic layouts), navigation (establishing the distance, navigability of the fairway)…

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Contents

Assessment strategies:

Assessing students' knowledge about the differential calculus during the lesson

MareMathics Teacher Toolkit and Digital Resources:

- Power point presentation to introduce differential calculus
- Videos
- GeoGebra

https://maremathics.pfst.hr/index.php/2021/09/07/calculus/
https://www.geogebra.org/t/calculus

Useful websites

https://www.wolframalpha.com/examples/mathematics/calculus-and-analysis/derivatives
https://www.symbolab.com/solver/calculus-calculator
https://www.scribd.com/document/472305804/Calculus-Volume-1-WEB-68M1Z5W-pdf

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LESSON FLOW

Time	Sequence	Content	Teacher activities	Student activities	Points for discussion
30 min	Starter/Introduction Presentation 6.1	Pre-teaching Introduction to the concept of derivative	Frontal then questioning, Motivation, Recall of main prerequisites	Active listening and contributing to questions	
110 min	Presentation 6.2 Exercises 1-6	Table of derivatives of elementary functions and basic rules of differentiation	Frontal then questioning, Motivation, Explains tasks, Discussion using solved examples	Active listening and contributing to questions, Solving exercises	Apply simple and complex derivation rules when solving tasks.
25 min	Presentation 6.3	Logarithmic differentiation	Frontal then questioning, Motivation, Explains tasks	Active listening and contributing to questions	Perform the derivation of the complex given function.
70 min	Presentation 6.4 Exercise 7	Derivation of the implicitly given function	Frontal then questioning, Motivation, Explains tasks, Discussion using solved examples	Active listening and contributing to questions, Solving exercises	Perform the derivation of the implicitly given function.
70 min	Presentation 6.5 Exercises 8-12	Derivation of the parametrically given function	Frontal then questioning, Motivation, Explains task and supports using Video Group work	Active listening and contributing to questions, Solving exercises	Perform the derivation of the parametrically given function. For example, if we know a parameterization of a given curve, how can we calculate the slope of a tangent line to the curve?
115 min	Presentation 6.6 Exercise 13-20	The tangent and normal lines to the graph	Frontal then questioning, Motivation, Explains tasks, Discussion using solved examples, Group work	Active listening and contributing to questions, Solving exercises	Explain the geometric interpretation of the derivative at a point.

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155 min	Presentation 6.7 Exercises 21-24	Application of derivatives to evaluate the limits of a functions	Frontal then questioning, Motivation, Solution of example, Explains task and supports using Video Group work	Active listening and contributing to questions, Solving exercises	L'Hospital's rule?
180 min	Presentation 6.8	Properties of continuous real function and graph sketching	Frontal then questioning, Motivation, Solution of example, Explains task and supports using Video Group work	Active listening and contributing to questions	Analyse the flow of an elementary function by using derivation, and sketch its graph.
90 min	Presentation 6.9 / Exercises	Exercises	Frontal then questioning, Motivation, Discussion using solved examples, Group work	Contributing to questions, Discussion, Solving exercises, Contributing to the solving process	More examples?
90 min	Presentation 6.10	Connections and applications (Related Rates and Optimization problem)	Frontal then questioning, Motivation, Explains tasks, Discussion using solved examples	Active listening and contributing to questions, Discussion, Contributing to the solving process	Derivatives are met in many problems in the maritime domain. More examples?
30 min.	Summary	Post-teaching	Posing the problem; recalling of knowledge; Solving Guides students to conclude the lessons Giving homeworks Helping students to solve more difficult exercises	Active listening and contributing to questions, Discussion, Contributing the solving process	Calculus can help us solve many types of real-world problems in maritime affairs.

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Lesson 1: Derivative

Lecture: 30 min	• **Whiteboard** • Lesson 6 https:// https://maremathics.pfst.hr/wp-content/uploads/2022/04/IO2-6-Calculus-1.pdf • The notion of derivation becomes clear with the help of examples. • Students can use this site (http://www.openstax.org/l/20_diffmicros) to explore graphs to see if they have a tangent line at a point. • The goal is to enable students not just to recognize concepts, but work with them in ways that will be useful in later courses and future careers.
Learning objectives	• **By the end of the lesson, students should understand the meaning of the derivative.** • **Recognize the meaning of the tangent to a curve at a point.** • **Calculate the slope of a tangent line.** • **Identify the derivative as the limit of a difference quotient.** • **Calculate the derivative of a given function at a point.**

Lesson 2: Table of derivatives of elementary functions and basic rules of differentiation

Lecture: 45 min Exercises: 65 min	• **Whiteboard** • Lesson 6 httpshttps:// https://maremathics.pfst.hr/wp-content/uploads/2022/04/IO2-6-Calculus-2.pdf • Exercises 1-6 (The teacher should possibly emphasize or repeat some details several times to make them easier for students to remember. This will make it easier for students to solve the exercises on their own.)
Learning objectives	• Estimate the derivative from a table of values. • Define the derivative function of a given function. • Apply the sum and difference rules to combine derivatives. • Use the product rule for finding the derivative of a product of functions. • Use the quotient rule for finding the derivative of a quotient of functions. • Extend the power rule to functions with negative exponents. • Combine the differentiation rules to find the derivative of a polynomial or rational function. • Find the derivatives of the standard trigonometric functions. • Apply the chain rule together with the power rule. • Recognize the chain rule for a composition of three or more functions. • Recognize the derivatives of inverse functions.

For exercise, a teacher can use GeoGebra Applets developed by MareMathics:

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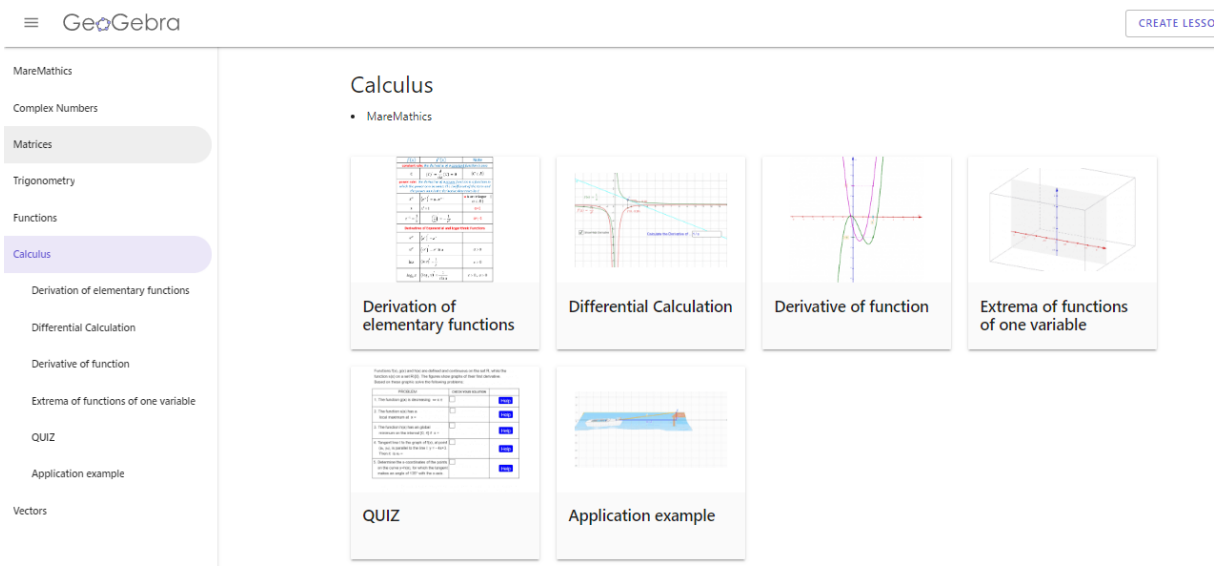

1. To determine derivatives of elementary functions
   .https://www.geogebra.org/m/zumaepzj
2. https://www.geogebra.org/m/cubzudht
3.

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Lesson 3: Logarithmic differentiation

Lecture: 25 min	• **Whiteboard** • **Lesson 6 https:// https://maremathics.pfst.hr/wp-content/uploads/2022/04/IO2-6-Calculus-3.pdf** • **The lesson guides students through the core concepts of calculus and helps them understand how those concepts apply.**
Learning objectives	• **Find the derivative of exponential functions.** • **Find the derivative of logarithmic functions.** • **Use logarithmic differentiation to determine the derivative of a function.**

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Lesson 4: Derivation of the implicitly given function

Lecture: 25 min Exercises: 45 min	• **Whiteboard** • Lesson 6 https:// https://marematics.pfst.hr/wp-content/uploads/2022/04/IO2-6-Calculus-4.pdf • In this section, we solve these problems by finding the derivatives of functions that define *y* implicitly in terms of *x*. • The notion of Implicit differentiation becomes clear with the help of examples and Exercise 7.
Learning objectives	• **Find the derivative of a complicated function by using implicit differentiation.** • **Use implicit differentiation to determine the equation of a tangent line.**

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Lesson 5: Derivation of the parametrically given function

Lecture: 20 min Exercises: 25 min Videos: 25 min	• **Whiteboard** • Lesson 6 https:// https://marematics.pfst.hr/wp-content/uploads/2022/04/IO2-6-Calculus-5.pdf • Our next step is to learn how to work with this concept in the context of calculus. For example, if we know a parameterization of a given curve, how can we calculate the slope of a tangent line to the curve? By the end of lesson 5, students should know this. • Exercises 8-12 • The Video "Derivatives" can help (see https://marematics.pfst.hr/?p=3542#derivatives).
Learning objectives	• **Determine the first derivatives of parametric equations.** • **Determine the equations of tangent lines to parametric curves.** • **Find the speed at any point in time for motion along a given parametric curve.**

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Lesson 6: The tangent and normal lines to the graph

Lecture: 25 min Exercises: 90 min	• **Whiteboard** • **Lesson 6 https:// https://maremathics.pfst.hr/wp-content/uploads/2022/04/IO2-6-Calculus-6.pdf** • **Exercises 13-20 (Students should have no problem learning this lesson. The teacher should possibly emphasize or repeat some details several times to make them easier for students to remember. This will make it easier for students to solve the exercises on their own.)**
Learning objectives	• **Calculate how to find the slope and equation of the tangent and normal to a curve at a given point using derivatives.**

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Lesson 7: Application of derivatives to evaluate the limits of a functions

Lecture: 45 min Exercises: 90 min Videos: 20 min	• **Whiteboard** • Lesson 6 https:// https://maremathics.pfst.hr/wp-content/uploads/2022/04/IO2-6-Calculus-7.pdf • Exercises 21-24 (In this section, we examine a powerful tool for evaluating limits. This tool, known as L'Hôpital's rule, uses derivatives to calculate limits. With this rule, we will be able to evaluate many limits we have not yet been able to determine. Instead of relying on numerical evidence to conjecture that a limit exists, we will be able to show definitively that a limit exists and to determine its exact value.) • The Video "Some derivation Applications" can help (see **https://maremathics.pfst.hr/?p=3542**).
Learning objectives	• **Recognize when to apply L'Hôpital's rule.** • **Identify indeterminate forms produced by quotients, products, subtractions, and powers, and apply L'Hôpital's rule in each case.**

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Lesson 8: Properties of continuous real function and graph sketching

Lecture: 90 min Exercises: 45 min Videos: 45 min	• **Whiteboard** • Lesson 6 https:// https://maremathics.pfst.hr/wp-content/uploads/2022/04/IO2-6-Calculus-8.pdf • After listening to the lecture, the students study the pdf file once again. The teacher then divides them into groups. Each group solves several tasks from the exercises. • Student can evaluate their knowledge solving the following Quiz **https://www.geogebra.org/m/seqqrc8r,** • Finally, tasks are compared and controlled. • The Video "Flow and Graph Functions" can help (see https://maremathics.pfst.hr/?p=3542#derivatives).
Learning objectives	• **Explain how the sign of the first derivative affects the shape of a function's graph.** • **State the first derivative test for critical points.** • **Use concavity and inflection points to explain how the sign of the second derivative affects the shape of a function's graph.** • **Explain the concavity test for a function over an open interval.** • **Explain the relationship between a function and its first and second derivatives.** • **State the second derivative test for local extrema.** • **Calculate the limit of a function as x increases or decreases without a bound.** • **Recognize a Vertical, Horizontal and Oblique asymptote on the graph of a function.** • **Estimate the end behaviour of a function as x increases or decreases without bound.** • **Analyse a function and its derivatives to draw its graph.**

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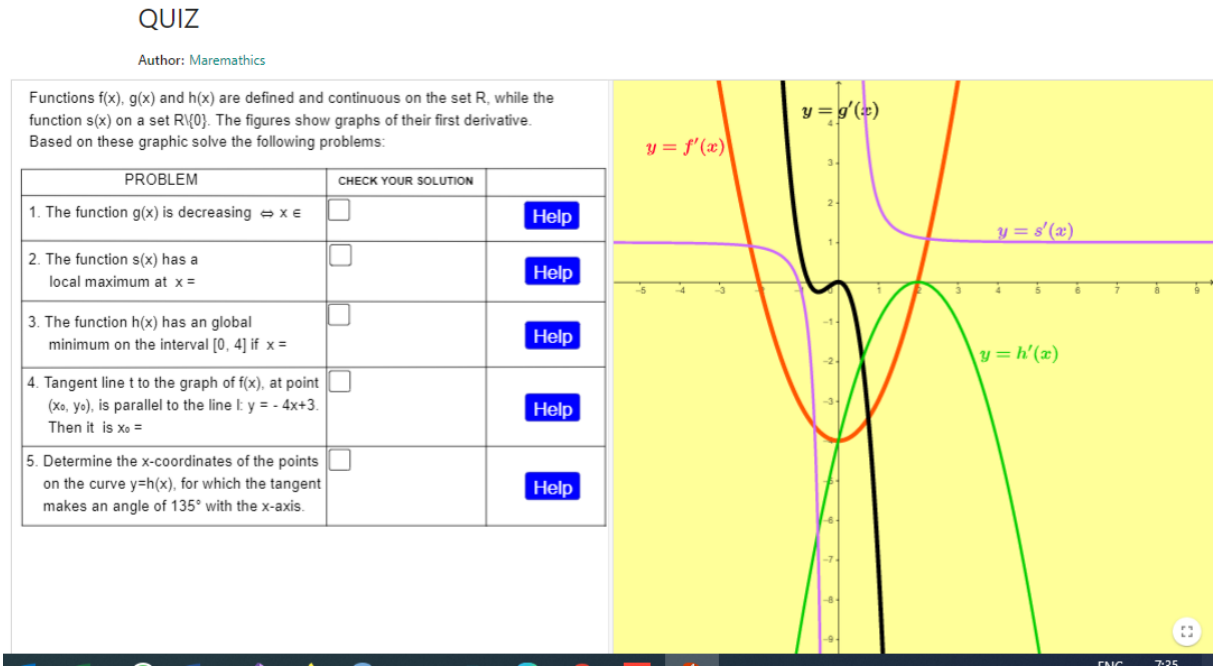
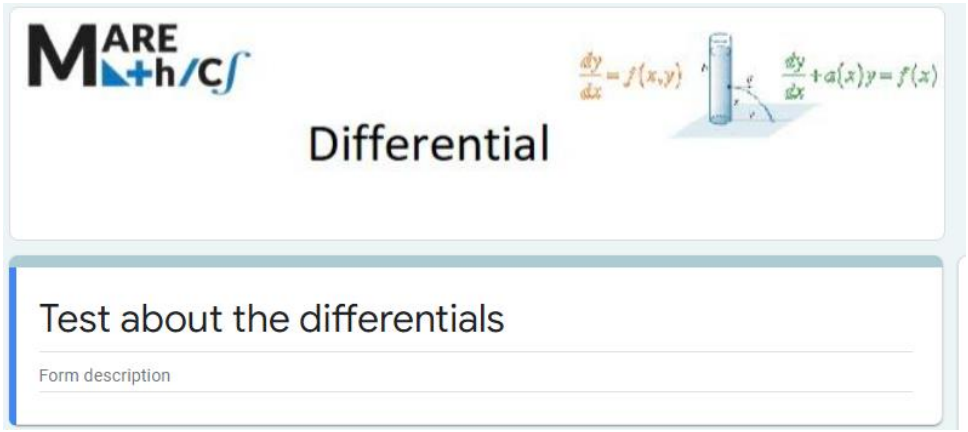
Note: Quiz https://www.geogebra.org/m/seqqrc8r



https://www.geogebra.org/m/seqqrc8r

Using HELP buttons students get graphical answer.

Lesson 9: Exercises

Exercises: 90 min	• **Whiteboard** • Lesson 6 https:// https://maremathics.pfst.hr/wp-content/uploads/2022/04/IO2-6-Calculus-9.pdf • The teacher can divide the students into groups so that each group solves one task from the exercises. If students have learning difficulties, the teacher can solve some more examples. • Solve the Test developed by MareMathics **https://forms.gle/8M7uRnh4vNNCx5fY6** 
Learning objectives	• **Combine the differentiation rules to find the derivative of a polynomial or rational function.** • **Apply the chain rule and the product/quotient rules correctly in combination when both are necessary.** • **Find the derivative of trigonometric, exponential and logarithmic functions.** • **Calculate the higher-order derivatives of the sine and cosine.** • **Calculate the derivative of inverse functions.** • **Use logarithmic differentiation to determine the derivative of a function.** • **Find the derivative of a complicated function by using implicit differentiation.** • **Find the derivative of a complicated function by using implicit and parametrically differentiation.** • **Use explicit, implicit and parametrically differentiation to determine the equation of a tangent and normal lines.** • **Identify indeterminate forms produced by quotients, products, subtractions, and powers, and apply L'Hôpital's rule in each case.**

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	• **Analyze a function and its derivatives to draw its graph.**

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Lesson 10: Connections and applications

10.1 Related Rates

Teachers can show and explain students the following example.

Example 1:

Ship A is 50 miles west of ship B. The ship A is sailing east at 10 knots, and the ship B is sailing south at 15 knots. Find the rate of change of the distance between the ships after 5 hours.

Example 2:

A ship sails according the law:

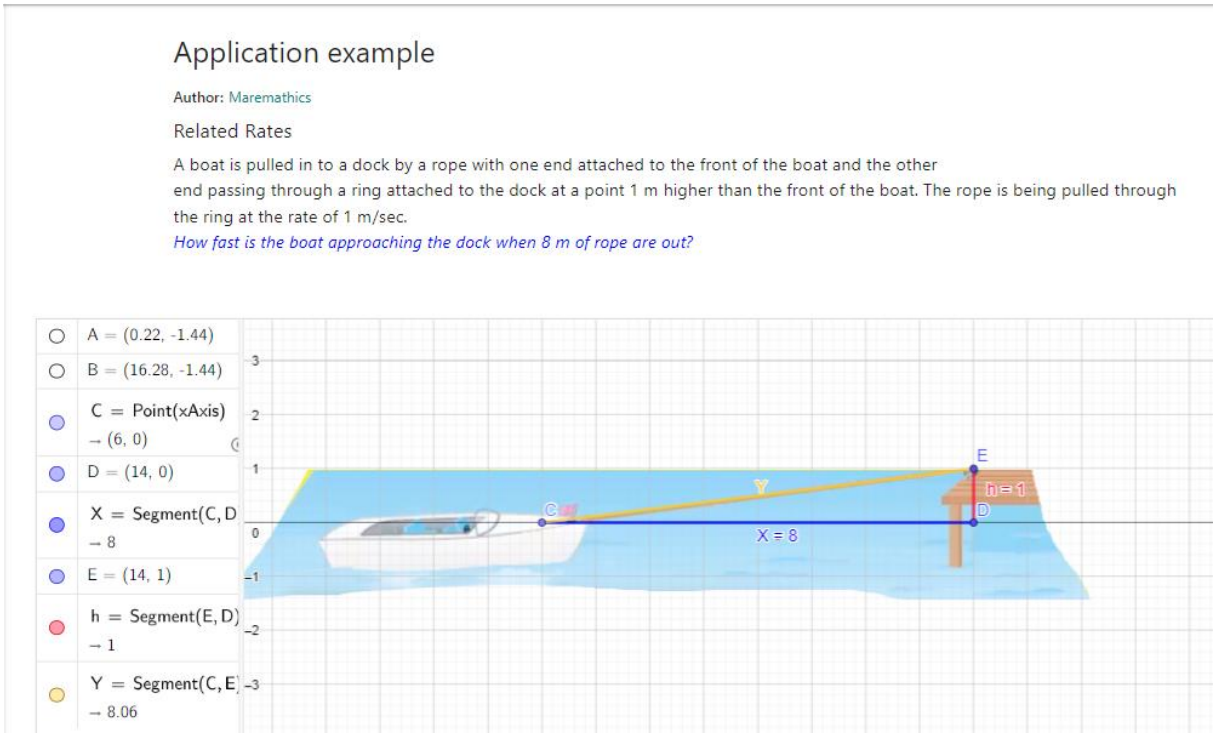
$$s = \left(1272.7 \cdot ln\frac{1 + 6 \cdot e^{0.055t}}{7} - 50t\right) \quad [m]$$

The start velocity of the ship according this voyage should be determined.

Example 3:

A boat is pulled in to a dock by a rope with one end attached to the front of the boat and the other end passing through a ring attached to the dock at a point 1 m higher than the front of the boat. The rope is being pulled through the ring at the rate of 1 m/sec. How fast is the boat approaching the dock when 8 m of rope are out?

To show the solution teachers can use GeoGebraApplet developed by MareMathics https://www.geogebra.org/m/dszb7cdj .



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It is possible the change the input of the task by input the new values in the left window of the applet.

10.2 Optimization problem (minimum, maximum)

Lecture: 45 min Exercises: 45 min	• **Whiteboard** • **Lesson 6 https:// https://marematics.pfst.hr/wp-content/uploads/2022/04/IO2-6-Calculus-10.pdf** • Calculus can help us solve many types of real-world problems in maritime affairs. Here we study several examples of related quantities that are changing with respect to time and we look at how to calculate one rate of change given another rate of change. Also, many important applied problems in maritime affairs involve finding the maximum or minimum value of some function like as the minimum time to rich the distance by a ship, the maximum profit, the minimum cost for doing a task, the maximum power of engines and so on. Many of these problems can be solved by finding the appropriate function and then using techniques of calculus to find the maximum or the minimum value required.
Learning objectives	• **Determine a new value of a quantity from the old value and the amount of change.** • **Apply rates of change to displacement, velocity, and acceleration of an object moving along a straight line.** • **Calculate the average rate of change and explain how it differs from the instantaneous rate of change.** • **Use the chain rule to find the rate of change of one quantity that depends on the rate of change of other quantities.** • **Find relationships among the derivatives in a given problem.** • **Students understand how Calculus apply to their lives and the world around them.**

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