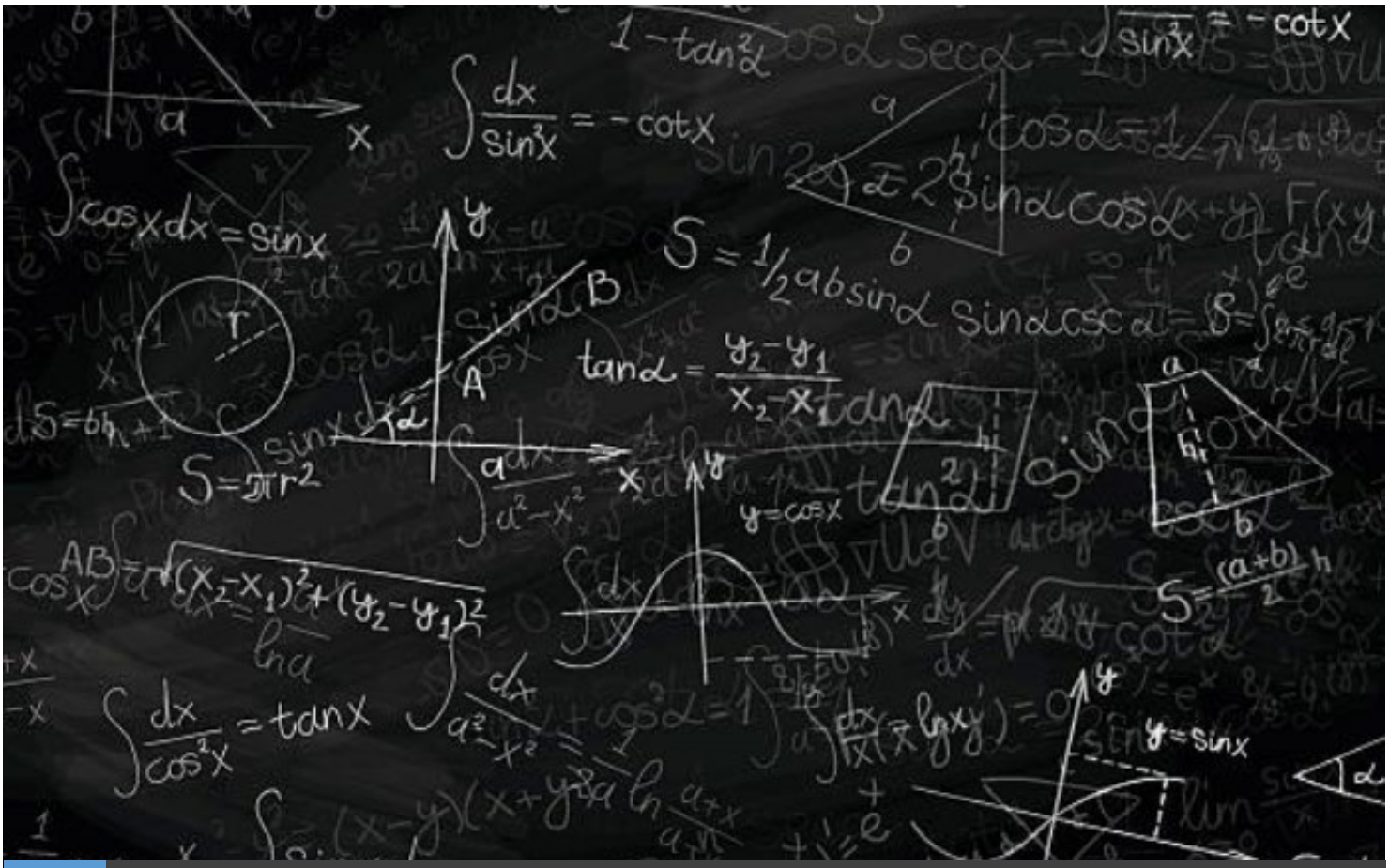
MARE Math/CS

Innovative Approach in Mathematical Education for Maritime Students



Teacher's Manual

Statistics and Probability

Author: Krzysztof Topolski

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MareMathics

Innovative Approach in Mathematical Education for Maritime Students

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https://maremathics.pfst.hr/

Manual for teachers

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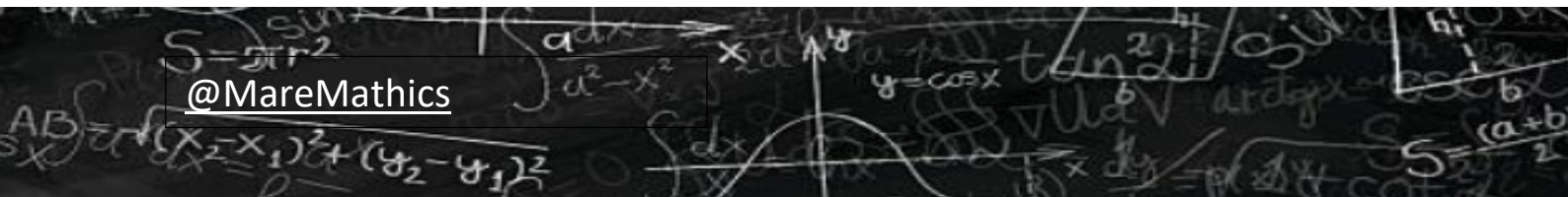
Reviewed by the teachers from Faculty of Maritime Studies in Split, Croatia



MARE Mathics

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Innovative Approach in Mathematical Education for Maritime Students

2019-1-HR01-KA203-061000

The Manual is the outcome of the collaborative work of all the Partners for the development of the MareMathics Project.

Partners in the project:

University of Split Faculty of Maritime Studies **CROATIA**

Tallinn University of Technology, Estonian Maritime Academy **ESTONIA**

Latvian Maritime Academy **LATVIA**

Polish Naval Academy Gdynia **POLAND**

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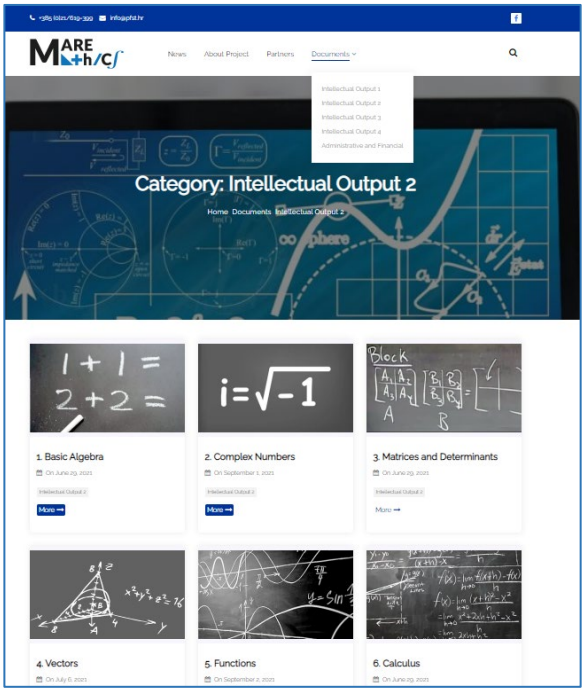
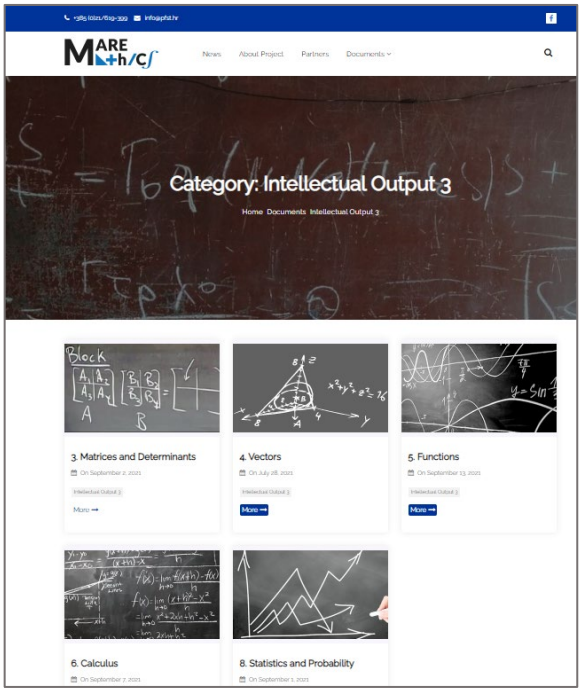
CONTENTS

Statistics and Probability: Teaching and Learning Plan

The goal of this material and related resources is to assist teachers in planning their lessons allowing achieving learning outcomes posted in the course's syllabus. It enables teachers to design student activities to encourage students to learn.

The resources are picked from project ***MareMathics*** and available on the https://maremathics.pfst.hr/.





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Lesson 1. Counting Principles

Name of Unit	Workload	Handbook
Counting Principles	**Lecture: 90 min** **Exercises: 90 min**	**Unit 9.1**

DETAILED DESCRIPTION

The unit of Counting principles presents the main counting principle such that the rule of product, formulas for the number of permutations or variations and for the number of combinations. Knowledge of these rules is very important in probabilistic methods we use to describe a real world. It has various applications in solving real life problems where the uncertainty is involved.

AIM: To learn and understand where different counting principles should be used.
To acquire also necessary skills in applying them.

Learning Outcomes:

At the end of this lecture, students should be able to

1. Explain and apply the rule of product
2. Explain and apply the definition of permutation and combination
3. Count the number of permutations and combinations
4. Solve basic combinatorial problems

Key words of this Unit:

Permutation, combinations, variation, factorial.

Previous knowledge of mathematics: Elementary mathematics: numbers and basic algebraic operations.

Relatedness with solving problems in the maritime field: The topic consisting counting principals is a prior knowledge in probabilistic and statistical methods that can be applied in maritime problems.

Contents:

- COUNTING PRINCIPLES
  - MULTIPLICATION PRINCIPLE
  - PERMUTATIONS
  - PERMUTATIONS OF N ELEMENTS TAKEN K AT TIME
  - PERMUTATION WITH REPETITION
  - COMBINATIONS

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Assessment strategies:

Evaluating student's activity during lesson.

Teacher Toolkit and Digital Resources:

- Lesson to explain counting principles
- Excel
- OpenOffice
- Work Sheets
- MareMathics Websites:
- **Lessons:** 9. Statistics and Probability https://maremathics.pfst.hr/?p=121
  - Videos: https://maremathics.pfst.hr/?cat=14
- Useful links
  - https://www.calculatorsoup.com/calculators/discretemathematics/combinations.php
  - https://www.openoffice.org
  - https://ocw.mit.edu/courses/mathematics/18-600-probability-and-random-variables-fall-2019/lecture-notes/

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Lesson: Counting Principles

LESSON FLOW					
Time	**Sequence**	**Content**	**Teacher activities**	**Student activities**	**Points for discussion**
5 min	Introduction	Pre-teaching	Moderator Motivation	Discussion	Where in real life we can use counting principles?
15 min	Presentation	The rule of product properties	Frontal explanation and solving examples	Active listening and contributing to questions	Are students able to recognize in which situation the rule of product is applied?
15 min	Presentation	PERMUTATIONS	Frontal explanation and solving examples	Active listening and contributing to questions	Are students able to use permutations in real life situations?
15 min	Presentation	PERMUTATIONS OF N ELEMENTS TAKEN K AT TIMES	Frontal explanation Discussion using solved examples	Active listening and contributing to questions	Are students able to compute factorial in the formula
15min	Presentation	Permutations with repetition	Frontal explanation Discussion using solved examples	Active listening and contributing to questions	Are students able to distinct permutations with repetition from other types of permutations
20min	Presentation	Combinations	Frontal explanation Discussion using solved examples	Active listening and contributing to questions	Are students able to compute binomial coefficients in the formula
5 min	Summary		Giving homework		

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SUGGESTED TEACHING STRATEGIES, INPUT AND RESOURCES

RESOURCES	• Whiteboard • Lesson 9.1. Counting Principles https://marematics.pfst.hr/wp-content/uploads/2021/09/IO2-8-Statistics-1.pdf • Excel spreadsheet: combinations https://marematics.pfst.hr/?p=4022
Learning objectives	By the end of the lesson: • ***all*** students should know basic counting principles • ***all*** students should be able to use in practice basic counting principles

A. A teacher presents students each counting principle.
Paragraphs: 9.1.1 – 9.1.6

https://marematics.pfst.hr/wp-content/uploads/2021/09/IO2-8-Statistics-1.pdf

B. The teacher shows students how to use each counting principle in practical examples.
Chapter 9.1, Examples: 9.1- 9.11

https://marematics.pfst.hr/wp-content/uploads/2021/09/IO2-8-Statistics-1.pdf

To illustrate Example 9.11 the teacher presents Excel spreadsheet: combinations

https://marematics.pfst.hr/?cat=16

To sum up the teacher presents a video: Permutations with Repetition

https://marematics.pfst.hr/index.php/2021/09/01/8-statistics-and-probability/

C. The teacher asks students to solve some exercises. He/she requires to remind the main rules presented at the beginning.

Paragraph 9.1.5, Exercise 9.2

https://marematics.pfst.hr/wp-content/uploads/2021/09/IO2-8-Statistics-1.pdf

C. The teacher introduces some practical applications of counting principles and explains students the possibility of meeting its in real life problems.
Exercise:
On how many ways can the ship from country A visit (only once) four harbours in countries B , C, D, E .

https://marematics.pfst.hr/wp-content/uploads/2021/09/IO2-8-Statistics-1.pdf

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Students Activity

- Teacher asks students to solve exercises to be sure they understand how to do their task and how to use the theory to real life problems.
- Students ask questions, solve their task on whiteboard or in the copybooks.
- Students work alone or in two-three persons groups, after solving problems students show their solutions to the rest of the whole group.
- Students watch and discuss or comment a short video on permutation.
- Students get some examples with answers to check and verify their solutions.
- Students get homework. They have to solve their tasks and show the solutions to teacher next lesson or send the solution to teacher by mail or via an educational platform.

A list of homework exercises

1. How many two-digit odd natural numbers greater than 30 are there?
2. How many three-digit even natural numbers less than 500 are there?
3. In how many ways can we arrange the letters of the word $MONTANA$?
4. There are seven harbours in a country A. In how many ways can four different ships dock there?
5. In how many ways can six coins be hidden in four boxes.
6. Six points lie on the circumference of a circle. How many of inscribed triangles can be drawn having these points as vertices?

Solutions can be found :

https://marematics.pfst.hr/wp-content/uploads/2022/06/Statistics-and-Probability-13.pdf

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Lesson 2. Probabillity

Name of Unit	Workload	Handbook
Probabillity	**Lecture: 90 min** **Exercises: 90 min**	**Units 9.2, 9.3, 9.4**

DETAILED DESCRIPTION

The unit of Probability presents the basic notions in probability theory such that: sample space, events, various definitions of probability, conditional probability. It also provides the basic formulas such that: total probability formula and Bayes rule formulas. Knowledge of these notions and rules is very important in probabilistic methods we use to describe a real world. It has various applications in solving real life problems where the uncertainty is involved also in maritime field.

AIM: To learn and understand what the probability is, and what we can count using the theory. To acquire also necessary skills in applying it.

Learning Outcomes:

At the end of this lecture, students should be able to

1. Explain and apply the definition of event and sample space
2. Explain and apply the classical definition of probability
3. Explain and apply the axiomatic definition of probability
4. Explain and apply the geometric definition of probability
5. Count the probability of events in real life problems
6. Explain and apply the definition of independent events
7. Explain and apply the definition of conditional probability
8. Solve the real-life problems using total probability formula and Bayes rule

Key words of this Unit:

Event, probability, conditional probability, Bayes rule.

Previous knowledge of mathematics: Elementary mathematics: numbers and basic algebraic operations.

Relatedness with solving problems in the maritime field: The topic consisting introduction to probability is a prior knowledge in probabilistic and statistical methods that can be applied in maritime problems. Firsts in a line examples are: risk of accident and ship insurance policy or weather forecasting .

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Contents:

- Events, probability
  - Sample space
  - Classical definition of probability
  - Axiomatic definition of probability
  - Geometric definition of probability
- Complete system of event, Bayes rule
  - Independent events
  - Conditional probability
  - Total probability formula
  - Bayes theorem

Assessment strategies:

Evaluating student's activity during lesson

Teacher Toolkit and Digital Resources:

- Presentation to explain Probability
- Excel
- OpenOffice
- Work Sheets
- MareMathics Websites:
  - **Lessons:** 9. Statistics and Probability https://maremathics.pfst.hr/?p=121
  - Videos: https://maremathics.pfst.hr/?cat=14
  - Presentation: https://maremathics.pfst.hr/?p=3708
- Useful links
  - https://www.calculatorsoup.com/calculators/discretemathematics/
  - https://www.openoffice.org
  - Lecture Notes | Uncertainty in Engineering | Civil and Environmental Engineering | MIT OpenCourseWare

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LESSON FLOW					
Time	**Sequence**	**Content**	**Teacher activities**	**Student activities**	**Points for discussion**
5 min	Introduction	Pre-teaching	Moderator Motivation	Discussion	Where in real life we can use counting principles ?
10 min	Presentation	Events	Frontal explanation and solving examples	Active listening and contributing to questions	Are students able to recognize what the event and sample space are?
10 min	Presentation	CLASSICAL DEFINITION OF PROBABILITY	Frontal explanation and solving examples	Active listening and contributing to questions	Are students able to use classical definition of probability in real life problems?
10 min	Presentation	AXIOMATIC DEFINITION OF PROBABILITY	Frontal explanation Discussion using solved examples	Active listening and contributing to questions	Are students able to understand what axiomatic definition of probability is for?
10min	Presentation	Geometric definition of probability	Frontal explanation Discussion using solved examples	Active listening and contributing to questions	Are students able to use geometric definition of probability in real life situations?
40min	Presentation	Complete systems of events, Bayes rule	Frontal explanation Discussion using solved examples	Active listening and contributing to questions	Are students able to use total probability formula and Bayes rule in real life problems ?
5 min	Summary		Giving homework		

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SUGGESTED TEACHING STRATEGIES, INPUT AND RESOURCES

Lesson : Probability

RESOURCES	• Whiteboard • Lesson https://marematics.pfst.hr/index.php/2021/07/07/8-statistics/ • https://marematics.pfst.hr/wp-content/uploads/2021/09/IO2-8-Statistics-2.pdf • https://marematics.pfst.hr/wp-content/uploads/2021/09/IO2-8-Statistics-3.pdf • https://marematics.pfst.hr/wp-content/uploads/2021/09/IO2-8-Statistics-4.pdf
Learning objectives	By the end of the lesson: • *all* students should know basic probability definitions and theorems • *all* students should be able to use in practice basic probability rules

A. Teacher presents students classical and axiomatic definition of probability

Chapter 9.2 page 11 and page 12

https://marematics.pfst.hr/wp-content/uploads/2021/09/IO2-8-Statistics-2.pdf

Teacher presents students geometric definition of probability

Chapter 9.3 page 14

https://marematics.pfst.hr/wp-content/uploads/2021/09/IO2-8-Statistics-3.pdf

B. Teacher shows students how to use classical definition of probability in practical examples.

Chapter 9.2, Example 9.14

https://marematics.pfst.hr/wp-content/uploads/2021/09/IO2-8-Statistics-2.pdf

Teacher shows students how to use geometric definition of probability in practical Examples.

Chapter 9.3, Examples: 9.15 - 9.16.

https://marematics.pfst.hr/wp-content/uploads/2021/09/IO2-8-Statistics-3.pdf

C. Teacher asks a student to solve some exercise, it requires to remind students the rules presented at the beginning

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Exercise

If the letters of word *algebra* are placed at random in a row, what is the probability that two successive letters will be a.

https://marematics.pfst.hr/wp-content/uploads/2021/09/IO2-8-Statistics-2.pdf

Exercise

Two ships have arriving time between 1 pm and 6 pm. Both ships must dock on the same berth. Once docked it takes each ship 50 minutes to restock and leave the dock. What is the probability that the ships won't have to wait for the berth?

https://marematics.pfst.hr/wp-content/uploads/2021/09/IO2-8-Statistics-3.pdf

D. Teacher presents students the following topics:
independent events: chapter 9.4 page 16,
conditional probability: chapter 9.4 page 17,
Bayes rule and total probability formula. chapter 9.4 page 18.

https://marematics.pfst.hr/wp-content/uploads/2021/09/IO2-8-Statistics-4.pdf

E. Teacher shows students how to use total probability formula and Bayes rule in practical examples.
Chapter 9.4, Examples 9.17-9.19

https://marematics.pfst.hr/wp-content/uploads/2021/09/IO2-8-Statistics-4.pdf

F. Teacher introduces some practical applications of the theory and explains students the possibility of using it in real life problems.

As an illustration teacher presents Example 9.20 and video: Bayes Rule

https://marematics.pfst.hr/index.php/2021/09/01/8-statistics-and-probability/

Students Activity

- Teacher asks students to solve exercises to be sure they understand how to do their task.
  Chapter 9.4 Exercises 9.3-9.5

  https://marematics.pfst.hr/wp-content/uploads/2021/09/IO2-8-Statistics-4.pdf

- Students ask questions, solve their task on whiteboard or in the copybooks.
- Students work alone or in two-three persons groups, after solving problem a student who first solve it shows his solution to the rest of the group.
- Students watch and discuss or comment a short video: Bayes Rule.

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https://maremathics.pfst.hr/index.php/2021/09/01/8-statistics-and-probability/

- Students get some examples with answers to check and verify their solutions.
- Students get homework. They have to solve their tasks and show the solution teacher next lesson or send the solution to teacher

A list of homework exercises

1. If the letters of word *about* are placed at random in a row, what is the probability that three successive letters will be vowels.

2. A couple wants to have three children. We suppose that the gender of the child is equally likely. Give the probabilities that:
   a) the couple has at least one boy,
   b) there is no girl older than a boy,
   c) the couple has exactly one girl.

3. Two ships A and B have arriving time between $\mathbf{1}$ pm and $\mathbf{5}$ pm. Both ships must dock on the same berth. Once docked, it takes each ship $\mathbf{30}$ minutes to restock and leave the dock. What is the probability
   a) that the ships won't have to wait for the berth?
   b) that the ship A won't have to wait for the berth?
   c) that the ship B will have to wait for the berth?

4. Suppose that two balls are drawn without replacement (the first ball is not replaced before the second is drawn) at random from a bag containing 4 red and 3 black balls. Let A be the event of drawing a red ball the first time, and B the event of drawing a red ball the second time. Are the events A and B independent?

5. Three identical bowls are labelled $1, 2, 3$. First bowl contains 3 red and 3 blue marbles. Second bowl contains 4 red and 2 blue marbles. Third bowl contains 1 red and 5 blue marbles. A bowl is randomly selected, and a marble is randomly selected from the bowl. a) What is the provability that a marble selected is blue? b) Given that a marble selected is red, what is the probability that bowl 2 was selected?

6. Suppose that the probability that John will solve a certain problem is $\frac{2}{3}$, that Mary will solve it is $\frac{3}{4}$ and that Bill will solve it is $\frac{1}{2}$. What is the probability
   a) that at least one person will solve it?
   b) that Mary and Bill will solve it but John will not?
   c) that John and Mary will solve it but Bill will not?
   d) that at least two people will solve it?

7. A box contains three coins: two regular coins and one fake two-headed coin $(P(H) = 1)$. You pick a coin at random and toss it.

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a) What is the probability that it lands heads up?
b) You pick a coin at random and toss it and get heads. What is the probability that it is the two-headed coin?

8. You toss a fair coin three times:
   a) What is the probability of three heads?
   b) What is the probability that you observe exactly one heads?
   c) Given that you have observed at least one heads, what is the probability that you observe at least two heads?

9. Articles coming through an inspection line are visually inspected by two successive inspectors. When a defective article comes through the inspection line, the probability that it gets by the first inspector is 0.1. The second inspector will miss five out of ten of the defective items that get past the first inspector. What is the probability that a defective item gets by both inspectors?

10. In an exam, two reasoning problems, 1 and 2, are asked. 35% students solved problem 1 and 15% students solved both the problems. How many students who solved the first problem will also solve the second one?

11. A certain disease has an incidence rate of 2%. Suppose that for some diagnostic test the false negative rate is 1% and false positive rate is 1%. Compute the probability that a person, chosen at random from the population:
    a) who tests positive actually has the disease.
    b) who tests negative actually has not the disease

Solutions can be found :

https://marematics.pfst.hr/wp-content/uploads/2022/06/Statistics-and-Probability-13.pdf

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Lesson 3. Discrete probability distribution

Name of Unit	Workload	Handbook
Discrete probability distribution	**Lecture: 90 min** **Exercises: 90 min**	**Unit 9.5**

DETAILED DESCRIPTION

The unit of Discrete probability distribution presents the basic notions related to discrete probability distribution such that: random variable, discrete random variable, cumulative distribution function. It also provides the basic definitions such that expected value (weighted average) and two the most important measures of dispersion: variance and standard deviation. In the unit we can also find examples which shows how to compute these values using given formulas. As the most important discrete probability distributions, binomial and Poisson distribution are considered. Knowledge of basic notions, definitions and formulas in the theory is very important in probabilistic methods we use to describe a real world. It has various applications in solving real life problems where the uncertainty is involved, also in maritime field.

AIM: To learn and understand what the discrete probability distribution is, and what we can count using the theory.
To acquire also necessary skills in applying it.

Learning Outcomes:

At the end of this lecture, students should be able to

1. Explain and apply the definition of discrete random variable
2. Explain and apply the definition of cumulative distribution function
3. Explain and apply the definition of expected value, variance and standard deviation
4. Explain and apply the definition of binomial distribution and Poisson distribution
5. Count the expected value, variance and standard deviation for discrete random variable
6. Solve the real-life problems using binomial and Poisson distributions

Key words of this Unit:
Discrete random variable, cumulative distribution function, expected value, variance, standard deviation, binomial distribution, Poisson distribution

Previous knowledge of mathematics: Elementary mathematics: numbers, basic algebraic operations, functions.

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Relatedness with solving problems in the maritime field: The topic consisting discrete random variable is a prior knowledge in probabilistic and statistical methods that can be applied in maritime problems.

Contents:

- **DISCRETE RANDOM VARIABLE,**
  - Expected value of discrete random variable
  - Variance and standard deviation of discrete random variable
- BINOMIAL DISTRIBUTION
- POISSON DISTRIBUTION

Assessment strategies:

Evaluating student's activity during lesson.

MareMathics Teacher Toolkit and Digital Resources:

- Presentation
- Excel
- OpenOffice
- Work Sheets
- MareMathics Websites:
  - **Lessons:** 9. Statistics and Probability https://maremathics.pfst.hr/?p=121
  - Videos: https://maremathics.pfst.hr/?cat=14
- Useful links
  - https://www.openoffice.org
  - Lecture Notes | Uncertainty in Engineering | Civil and Environmental Engineering | MIT OpenCourseWare

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Lesson : Discrete random variable

LESSON FLOW					
Time	**Sequence**	**Content**	**Teacher activities**	**Student activities**	**Points for discussion**
5 min	Introduction	Pre-teaching	Moderator Motivation	Discussion	Where in real life we can use discrete random variable?
20 min	Presentation	The definition of discrete random variable	Frontal explanation and solving examples	Active listening and contributing to questions	Are students able to recognize what the discrete random variable is, and what we use it for?
20 min	Presentation	Expected value, variance and standard deviation of the discrete random variable	Frontal explanation and solving examples	Active listening and contributing to questions	Are students able to compute expected value, variance and standard deviation of discrete random variable?
20 min	Presentation	Binomial distribution	Frontal explanation Discussion using solved examples	Active listening and contributing to questions	Are students able to apply binomial distribution?
20min	Presentation	Poisson distribution	Frontal explanation Discussion using solved examples	Active listening and contributing to questions	Are students able to apply Poisson distribution?
5 min	Summary		Giving homework		

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SUGGESTED TEACHING STRATEGIES, INPUT AND RESOURCES

RESOURCES	• **Whiteboard** • Lesson https://marematics.pfst.hr/index.php/2021/07/07/8-statistics/ • https://marematics.pfst.hr/wp-content/uploads/2021/09/IO2-8-Statistics-5.pdf • https://marematics.pfst.hr/wp-content/uploads/2021/09/IO2-8-Statistics-6.pdf • https://marematics.pfst.hr/wp-content/uploads/2021/09/IO2-8-Statistics-12.pdf
Learning objectives	By the end of the lesson: • *all* students should know what a discrete random variable is and how to compute expected value, variance and standard deviation • *all* students should be able to use in practice binomial distribution and Poisson distribution

A. Teacher presents students the definition of discrete random variable and its distribution.

Chapter 9.5

https://marematics.pfst.hr/wp-content/uploads/2021/09/IO2-8-Statistics-5.pdf

B. Teacher shows students how to compute expected value, variance and standard deviation.

Chapter 9.5, Example 9.23-9.24

https://marematics.pfst.hr/wp-content/uploads/2021/09/IO2-8-Statistics-5.pdf

C. Teacher asks a student to solve some exercises,

Exercise:
The probability distribution for a random variable X is given in table.

x_i	-3	-1	0	2	3
p_i	0.1	0.4	0.2	0.2	0.1

Find the mean, variance, and standard deviation of X.

https://marematics.pfst.hr/wp-content/uploads/2022/06/Statistics-and-Probability-13.pdf

D. Teacher presents student the definition of binomial distribution and Poisson distribution

Chapter 9.6

https://marematics.pfst.hr/wp-content/uploads/2021/09/IO2-8-Statistics-6.pdf

E. Teacher presents students how to apply binomial distribution and Poisson distribution

Chapter 9.6 Examples 9.26-9.27

https://marematics.pfst.hr/wp-content/uploads/2021/09/IO2-8-Statistics-6.pdf

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F. Teacher introduces some practical applications of the theory and explains students the possibility of using it in real life problems.

https://maremathics.pfst.hr/wp-content/uploads/2021/09/IO2-8-Statistics-12.pdf

Students Activity

- Teacher asks students to solve exercises to be sure they understand how to do their task and how to use exponentials to real life problems.

https://maremathics.pfst.hr/wp-content/uploads/2022/06/Statistics-and-Probability-13.pdf

- Students ask questions, solve their task on whiteboard or in the copybooks.
- Students work alone or in two-three persons groups, after solving problems students show their solutions to the rest of the whole group.
- Students get homework. They have to solve their tasks and show the solutions to teacher next lesson or send the solution to teacher by mail or via an educational platform .

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A list of homework exercises

1. The probability distribution for a random variable X is given in table.

x_i	-3	-1	0	2	3
p_i	0.1	0.4	0.2	0.2	0.1

Find the mean, variance, and standard deviation of X.

2. A supervisor in a factory has three men and three women working for him. He wants to choose two workers for a special job. He decides to select the two workers at random. Let Y denote the number of women in his selection. Find the probability distribution for Y.

3. Each of three balls are randomly placed into one of three bowls. Find the probability distribution for Y = the number of empty bowls.

4. A balanced coin is tossed three times. Let Y equal the number of heads observed.

   a) Calculate the probabilities associated with $Y = 0, 1, 2$, and 3.

   b) Construct a probability distribution table.

   c) Find the expected value and standard deviation of Y.

5. An insurance company issues a one-year $\$2000$ policy insuring against an occurrence A that historically happens to 1 out of every 100 owners of the policy. How much should the company charge for the policy if it requires that the expected profit per policy be $\$75$?

6. A basketball player takes 4 independent freethrows with a probability of 0.7 of getting a basket on each shot. Let Y = the number of baskets he gets. Find the probability distribution for a random variable Y. Find the probability that he gets at least 3 baskets.

7. Suppose that a radio contains six transistors, two of which are defective. Three transistors are selected at random, removed from the radio, and inspected. Let X equal the number of defectives observed. Find the probability distribution for X.

8. Suppose that two balls are drawn with replacement (the first ball is replaced before the second is drawn) at random from a bag containing 5 red and 3 black balls. Let X is equal to the number of black balls drown. Find the probability distribution for a random variable X .

9. Suppose that two balls are drawn with no replacement at random from a bag containing 5 red and 3 black balls. Let X is equal to the number of black balls drown. Find the probability distribution for a random variable X .

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10. The probability distribution for a random variable X is given in table.

x_i	0	2	4	5	6
p_i	$\frac{1}{3}$	$\frac{1}{12}$	$\frac{1}{3}$	$\frac{1}{6}$	$\frac{1}{12}$

a) Find the cumulative distribution function of X. Sketch the graph.
b) Calculate: i) $P(2 \leq X \leq 4)$ ii) $P(0 < X < 4)$ iii) $P(X > 1)$.

11. A football player takes three independent penalties with a probability $\frac{4}{5}$ of scoring a goal on each shot. Let X be the number of goals he scores. Find the probability distribution for a random variable X. Find the expected number of goals.

12. Some complex system has an average failure rate λ=0.005 lamp failures per hour. What is the reliability for a 60 days period if the number of lamp failures cannot exceed 2?

Solutions can be found :
https://marematics.pfst.hr/wp-content/uploads/2022/06/Statistics-and-Probability-13.pdf

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Lesson 4. Continuous probability distribution

Name of Unit	Workload	Handbook
Continuous probability distribution	**Lecture: 90 min** **Exercises: 90 min**	**Unit 9.5 - 9.8**

DETAILED DESCRIPTION

The unit of Continuous probability distribution presents the basic notions related to continuous probability distribution such that: random variable, discrete random variable, cumulative distribution function, density function. It also provides the basic definitions such that expected value (weighted average) and two the most important measures of dispersion: variance and standard deviation. In the unit we can find also examples which shows how to compute these values using given formulas. As the most important continuous probability distributions, uniform and normal distribution are considered. Knowledge of basic notions, definitions and formulas in the theory is very important in the probabilistic methods we use to describe a real world. It has various applications in solving real life problems where the uncertainty is involved, also in maritime field.

AIM: To learn and understand what the continuous probability distribution is, and what we can count using the theory.
To acquire also necessary skills in applying it.

Learning Outcomes:

At the end of this lecture, students should be able to

1. Explain and apply the definition of continuous random variable
2. Explain and apply the definition of cumulative distribution function and density function for continuous random variable
3. Explain and apply the definition of expected value, variance and standard deviation for continuous random variable
4. Explain and apply the definition of normal distribution and standard normal distribution
5. Count the expected value, variance and standard deviation for continuous random variable
6. Solve the real-life problems using normal and standard normal distributions

Key words of this Unit:

Continuous random variable, cumulative distribution function, density function, expected value, variance, standard deviation, normal distribution

Previous knowledge of mathematics: Calculus, including integrals

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Relatedness with solving problems in the maritime field: The topic consisting continuous random variable is a basic knowledge in probabilistic and statistical methods that can be applied in maritime problems.

Contents:

CONTINUOUS RANDOM VARIABLE,

- Cumulative distribution function and density function for continuous random variable
- Expected value of continuos random variable
- Variance and standard deviation of continuous random variable

NORMAL DISTRIBUTION

Assessment strategies:

Evaluating student's activity during lesson.

Teacher Toolkit and Digital Resources:

- Presentation
- Excel
- OpenOffice
- Work Sheets
- MareMathics Websites:
  - **Lessons:** 9. Statistics and Probability https://maremathics.pfst.hr/?p=121
  - Videos: https://maremathics.pfst.hr/?cat=14
- Useful links
  - https://www.openoffice.org
  - Lecture Notes | Uncertainty in Engineering | Civil and Environmental Engineering | MIT OpenCourseWare

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Lesson : Continuous random variable

LESSON FLOW					
Time	**Sequence**	**Content**	**Teacher activities**	**Student activities**	**Points for discussion**
5 min	Introduction	Pre-teaching	Moderator Motivation	Discussion	Where in real life we can use continuous random variable?
20 min	Presentation	The definition of continuous random variable. Example	Frontal explanation and solving examples	Active listening and contributing to questions	Are students able to recognize what the continuous random variable is, and what we use it for?
20 min	Presentation	Expected value, variance and standard deviation of the continuous random variable	Frontal explanation and solving examples	Active listening and contributing to questions	Are students able to compute expected value, variance and standard deviation of continuous random variable?
40 min	Presentation	Normal distribution	Frontal explanation Discussion using solved examples	Active listening and contributing to questions	Are students able to apply normal distribution?
5 min	Summary		Giving homework		

SUGGESTED TEACHING STRATEGIES, INPUT AND RESOURCES

RESOURCES	• **Whiteboard** • Lesson https://marematics.pfst.hr/index.php/2021/07/07/8-statistics/ • https://marematics.pfst.hr/wp-content/uploads/2021/09/IO2-8-Statistics-7.pdf • https://marematics.pfst.hr/wp-content/uploads/2021/09/IO2-8-Statistics-8.pdf
Learning objectives	By the end of the lesson: • *all* students should know what a continuous random variable is and how to compute expected value, variance and standard deviation • *all* students should be able to use in practice normal distribution and uniform distribution

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A. Teacher presents students with the definition of continuous random variable.

Chapter 9.7

https://marematics.pfst.hr/wp-content/uploads/2021/09/IO2-8-Statistics-5.pdf

B. Teacher shows students how to compute an expected value, variance and standard deviation for some continuous random variable.

Chapter 9.7, Example 9.28

https://marematics.pfst.hr/wp-content/uploads/2021/09/IO2-8-Statistics-7.pdf

C. Teacher asks a student to solve some exercises,

Chapter 9.7, Exercise 9.6

https://marematics.pfst.hr/wp-content/uploads/2021/09/IO2-8-Statistics-7.pdf

D. Teacher presents to students the definition of normal distribution and standard normal distribution

Chapter 9.8

https://marematics.pfst.hr/wp-content/uploads/2021/09/IO2-8-Statistics-8.pdf

E. Teacher presents to students how to apply normal distribution

Chapter 9.8 , Example 9.29

https://marematics.pfst.hr/wp-content/uploads/2021/09/IO2-8-Statistics-8.pdf

F. Teacher introduces some practical applications of the theory and explains to students the possibility of using it in real-life problems.

Chapter 9.8 , Exercise 9.29

https://marematics.pfst.hr/wp-content/uploads/2021/09/IO2-8-Statistics-8.pdf

Students Activity

- Teacher asks students to solve exercises to be sure they understand how to do their task and how to use the theory to real life problems.
  Chapter 8.7-9.8 Exercises 9.6-9.7
- Students ask questions, solve their task on whiteboard or in the copybooks.
- Students work alone or in two-three persons groups, after solving problems students show their solutions to the rest of the whole group.
- Students get homework. They have to solve their tasks and show the solutions to teacher next lesson or send the solution to teacher.

Homework exercises:

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1. A variable X has a normal distribution with a mean of 10 and a standard deviation of 2. One score is randomly sampled. What is the probability that it is between 11 and 12?
2. A variable X has a standard normal distribution. One score is randomly sampled. What is the probability that it is between 1 and 2?

Solutions can be found :

https://marematics.pfst.hr/wp-content/uploads/2022/06/Statistics-and-Probability-13.pdf

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Lesson 5. Statistics

Name of Unit	Workload	Handbook
Statistics	**Lecture: 90 min** **Exercises: 90 min**	**Units 9.9 – 9.12**

DETAILED DESCRIPTION

The unit of Statistics presents the basic notions in statistics theory such that: sample, sample mean, sample variance, sample standard deviation. It also provides the examples showing how to compute these values. In the second part the problem of testing statistical hypotheses is considered. Two models of such testing are given, and two types of possible errors are defined. The last part is devoted to correlation and linear regression, which we can define and compute in case when we have two data samples. Knowledge of the given definitions, rules and testing methods is very important in statistical techniques we use to describe a real world. It has various applications in solving real life problems where the uncertainty is involved, also in maritime field.

AIM: To learn and understand what the statistic is, and what we can count using the theory. To acquire also necessary skills in applying it.

Learning Outcomes:

At the end of this lecture, students should be able to

1. Explain and apply the basic definitions and notations of statistics
2. Count sample mean, sample variance, sample standard deviation
3. Explain and apply how to test statistical hypotheses
4. Explain and apply the definition of correlation and linear regression
5. Count the correlation and linear regression of two data samples
6. Solve the real-life problems using basic statistical methods

Key words of this Unit:

Sample, mean, variance, standard deviation, correlation, regression , statistical hypothesis.

Previous knowledge of mathematics: Elementary mathematics: numbers, basic algebraic operations, functions.

Relatedness with solving problems in the maritime field: The topic consisting statistic is a prior knowledge in statistical methods which can be applied in maritime problems. One of the most important are numerous maritime transport statistics. For instance, we use statistics to investigate liner shipping connectivity, time spent at port in relation to a market segment, port container traffic, etc.

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Contents:

Assessment strategies:

Evaluating student's activity during lesson.

Teacher Toolkit and Digital Resources:

- Presentation
- Excel
- OpenOffice
- Work Sheets
- Websites:
  - https://maremathics.pfst.hr/index.php/2021/07/07/8-statistics/
  - https://www.openoffice.org
  - http://onlinestatbook.com/2/calculators/inverse_normal_dist.html
  - https://www.statology.org/inverse-t-distribution-calculator

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Lesson : Statistics

LESSON FLOW					
Time	**Sequence**	**Content**	**Teacher activities**	**Student activities**	**Points for discussion**
5 min	Introduction	Pre-teaching	Moderator Motivation	Discussion	Where in real life we can use statistics?
20 min	Presentation	Basic statistics definition and notations	Frontal explanation and solving examples	Active listening and contributing to questions	Are students able to recognize what the sample mean, variance and standard deviation is and what we use it for?
40 min	Presentation	TESTING OF STATISTICAL HYPOTHESES	Frontal explanation and solving examples	Active listening and contributing to questions	Are students able to test statistical hypotheses using Model I and Model II?
20 min	Presentation	CORRELATION AND REGRESSION	Frontal explanation Discussion using solved examples	Active listening and contributing to questions	Are students able to compute correlation and linear regression?
5 min	Summary		Giving homework		

SUGGESTED TEACHING STRATEGIES, INPUT AND RESOURCES

RESOURCES	• **Whiteboard** • Lesson https://marematics.pfst.hr/index.php/2021/07/07/8-statistics/ • https://marematics.pfst.hr/wp-content/uploads/2021/09/IO2-8-Statistics-9.pdf • https://marematics.pfst.hr/wp-content/uploads/2021/09/IO2-8-Statistics-10.pdf • https://marematics.pfst.hr/wp-content/uploads/2021/09/IO2-8-Statistics-11.pdf
Learning objectives	By the end of the lesson: • *all* students should know how to compute sample mean, variance and standard deviation. • *all* students should be able to test statistical hypotheses using Model I and Model II

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	• ***all*** students should know how to compute correlation and linear regression of two given samples .

A. Teacher presents to students basic definitions of statistics.

Chapter 9.9

https://marematics.pfst.hr/wp-content/uploads/2021/09/IO2-8-Statistics-9.pdf

B. Teacher shows students how to compute sample mean, sample variance and sample standard deviation.

Chapter 9.9, Examples 9.30-9.33

https://marematics.pfst.hr/wp-content/uploads/2021/09/IO2-8-Statistics-9.pdf

D. Teacher asks students to solve some exercises,
Exercise:
For a given sample:: $1, 3, 8, 4, 6, 9, 3, 7, 4, 4$ calculate: mean, variance and standard deviation. OpenOffice spreadsheet: mean_variance

https://marematics.pfst.hr/?cat=16

E. Teacher presents to students how to test statistical hypotheses and introduces some practical applications of the theory
Chapter 9.10 , Examples 9.34-9.35

https://marematics.pfst.hr/wp-content/uploads/2021/09/IO2-8-Statistics-10.pdf

E. Teacher presents to students how to compute covariance, correlation of two data samples

Chapter 9.11, Examples 9.36-9.37

https://marematics.pfst.hr/wp-content/uploads/2021/09/IO2-8-Statistics-11.pdf

and linear regression

Chapter 9.11.1, Example 9.38

https://marematics.pfst.hr/wp-content/uploads/2021/09/IO2-8-Statistics-11.pdf

OpenOffice worksheet: regression

https://marematics.pfst.hr/?cat=16

F. Teacher introduces some practical applications of the theory and explains students the possibility of using it in real life problems. OpenOffice worksheet: income_empl

https://marematics.pfst.hr/?cat=16

Students Activity

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- Teacher asks students to solve exercises to be sure they understand how to do their task and how to apply the theory in real life problems.

Chapter 9.13 , Exercise 35

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- Students ask questions, solve their task on whiteboard or in the copybooks.
- Students work alone or in two-three persons groups, after solving problems students show their solutions to the rest of the whole group.
- Students get some examples with answers to check and verify their solutions.

Chapter 9.11.1 , Exercise 9.8

https://marematics.pfst.hr/wp-content/uploads/2021/09/IO2-8-Statistics-11.pdf

- Students get homework. They have to solve their tasks and show the solutions to teacher next lesson or send the solution to teacher.

Homework exercises:

1. In a biochemical experiment, lifespan of certain organisms was measured. Distribution of that time can be assumed as normal. 8 measurements were taken, and the results were (in hours): $4.7;\ 5.3;\ 4.0;\ 3.8;\ 6.2;\ 5.5;\ 4.5;\ 6.0.$ Assuming the significance level $\alpha = 0{,}05$, assess the hypothesis that the average lifespan of these organisms is 4.0 hours.

2. Find a linear regression function for data: x= -1,1,2, y= 1,-2,2.

Solutions can be found :

https://marematics.pfst.hr/wp-content/uploads/2022/06/Statistics-and-Probability-13.pdf

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