Teacher's Manual

MATRICES AND DETERMINANTS

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MareMathics

Innovative Approach in Mathematical Education for Maritime Students

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https://maremathics.pfst.hr/

Manual for teachers

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Reviewed by Agata Załęska-Fornal, Poland
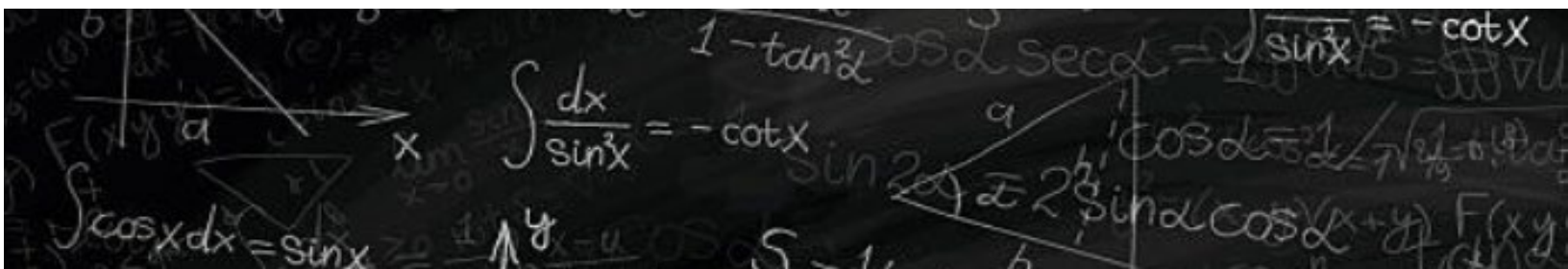
The Manual is the outcome of the collaborative work of all the Partners for the development of the MareMathics Project.

Partners in the project:
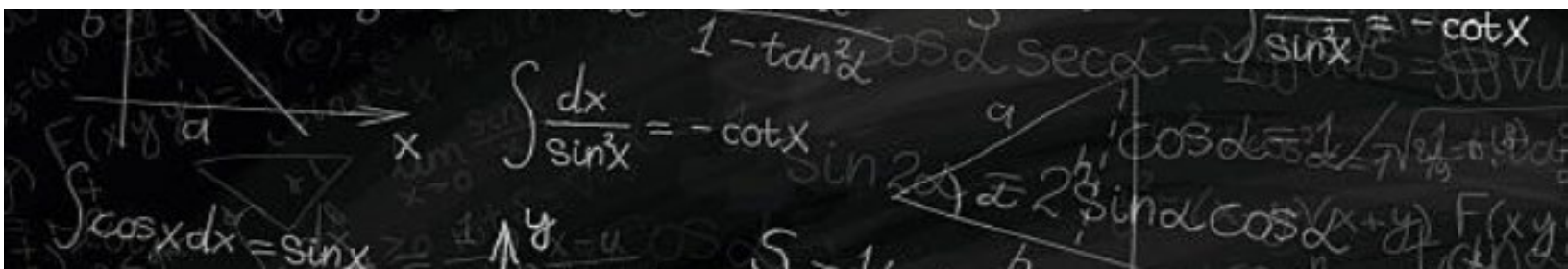
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CONTENTS

MATRICES AND DETERMINANTS: Teaching and Learning Plan

The goal of this material and related resources is to assist teachers in planning their lessons allowing achieving learning outcomes posted in the course's syllabus. It enables teachers to design student activities to encourage students to learn.

The resources are picked from project **MareMathics** and available on the https://maremathics.pfst.hr/.
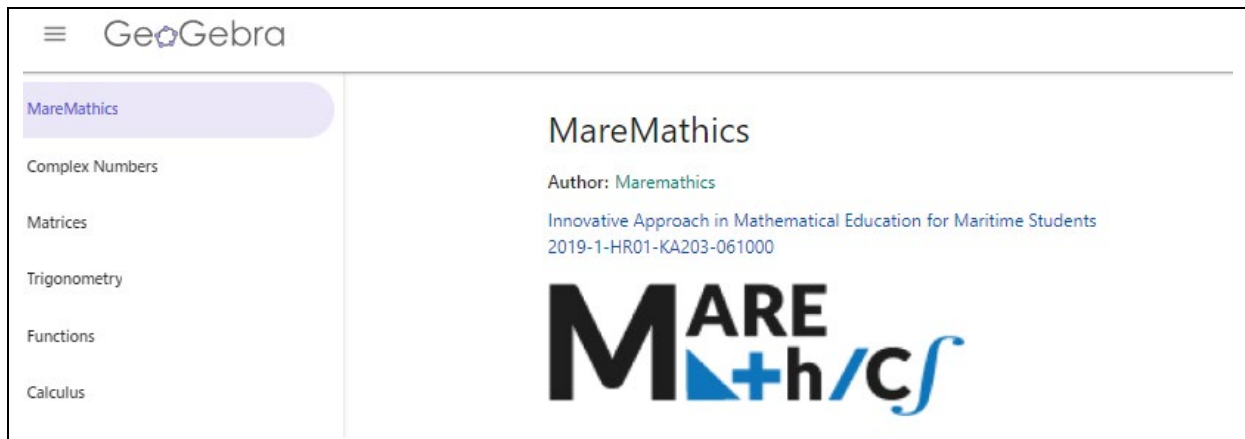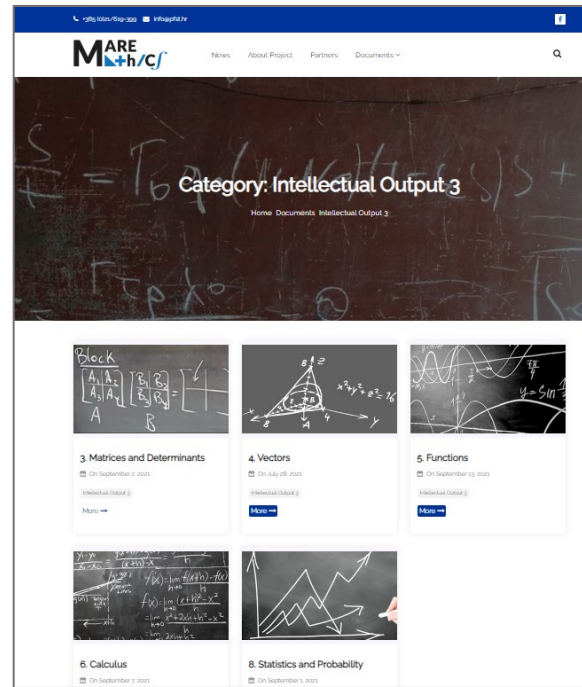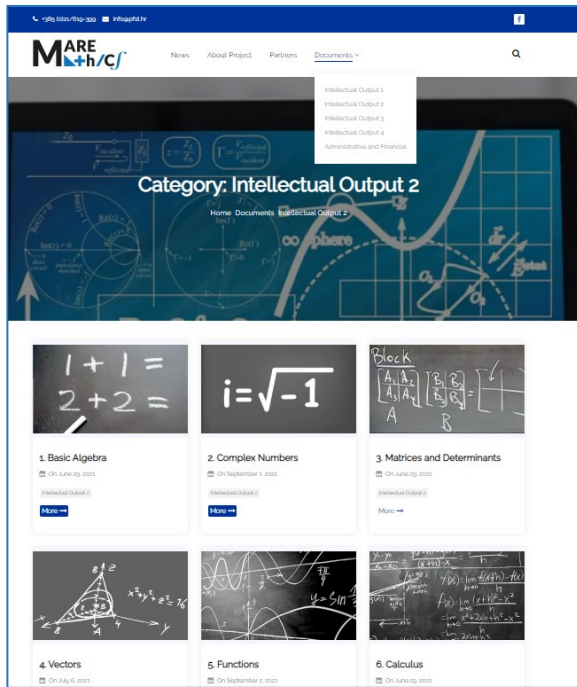
Lesson 1. The Matrix

Name of Unit	Workload	Handbook
Matrices	**Lecture: 90 min** **Exercises: 90 min**	**Unit 3. Matrices and determinants**

DETAILED DESCRIPTION

This chapter introduces the concept of matrix. Firstly, some operations with matrices are defined, then the determinant of the square matrix and the inverse matrix of the regular matrix. After that, it is demonstrated how to solve matrix equations and how a system of linear equations can be solved using matrices. Each topic is covered in detail, and in addition to some solved examples, it also contains exercises. At the end of the chapter is a knowledge test. Additional applications in MATLAB, Excel and Geogebra can be used to check the obtained solutions or to make faster calculations (if the procedure for solving exercises is not required).

AIM:
Learn how to calculate the determinant, inverse matrix and how to solve a system of linear equations using matrices.

Learning Outcomes:

1. Know how and when arithmetic operations with matrices are defined.

2. Calculate the determinant using the basic properties of the determinant.

3. Determine the inverse matrix using the Gauss-Jordan method.

4. Solve a linear system using matrices.

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Key words of this Unit:

Matrix, determinant, inverse matrix, matrix rank, linear algebraic equation

Previous knowledge of mathematics: The student should know the basic arithmetic operations (addition, subtraction, multiplication, and division) with real numbers.

Application:

- in geometry

- in processing of digital photography

- a model of consumer preference

- encryption and decoding of messages in cryptography

- analysis of an economic system

- regression line with the least square deviation from the given data set

- a problem of transport and distribution

- travelling salesman problem

Contents:

3.1   Matrix

3.2   Matrix arithmetic

3.3   Determinant of a square matrix

3.4   Inverse matrix

3.5   Matrix equations

3.6   Matrix rank

3.7   Systems of linear algebraic equations

3.8   Some examples of matrix applications

Assessment strategies:

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MarMathics Teacher Toolkit and Digital Resources:

- Powerpoint presentation to introduce and work with matrices

Geogebra Learning Tools

- Videos
- Quizzes /WorkSheets

- These resources are for revision at the end of the lesson. They could also be used to introduce some of the concepts, but you may need to split the work into sections and provide extra examples and problems.

LESSON FLOW

Time	Sequence	Content	Teacher activities	Student activities	Points for discussion
15 min 10 min	Starter/Introduction Presentation 3.1 Exercises 1-5	What is matrix? Some special types of matrices	Frontal then questioning Group work	Active listening and contributing to questions Solving exercises	
15 min 15 min	Presentation 3.2 Exercises 6-10	Matrix arithmetic	Frontal then questioning Group work	Active listening and contributing to questions Solving some exercises	Practical applications of matrix arithmetic
30 min 30 min	Presentation 3.3 Exercises 11-12	Determinant of a square matrix	Frontal Questioning Explains tasks	Active listening Contributing Solving some exercises	
30 min 30 min	Presentation 3.4 Exercise 14	Inverse matrix	Frontal Discussion using solved examples	Active listening and contributing to questions Solving exercise	Gauss-Jordan method
20 min 30 min	Presentation 3.5 Exercises 15-20	Matrix equations	Frontal Discussion using solved examples Group work	Active listening Discussion Solving some exercises	
45 min 30 min	Presentation 3.6 Exercise 21	Matrix rank	Frontal Discussion using solved examples	Active listening Discussion Solving exercise	Gauss method
60 min 60 min	Presentation 3.7 Exercises 22-27	Systems of linear algebraic equations	Frontal Questioning Discussion using solved examples	Active listening Discussion Solving some exercises	When the system has no solution?
30 min	Presentation 3.8	Some examples of matrix application	Frontal Discussion using solved examples	Active listening Discussion	More examples?
30 min	Summary	Post-teaching	Guides students to conclude the lessons Giving homeworks Helping students to solve more difficult exercises		

SUGGESTED TEACHING STRATEGIES, INPUT AND RESOURCES

Resources	• **Whiteboard** • **Lesson 1** [**https://maremathics.pfst.hr/wp-content/uploads/2021/09/IO2-3-Matrices-and-Determinants-1.pdf**](https://maremathics.pfst.hr/wp-content/uploads/2021/09/IO2-3-Matrices-and-Determinants-1.pdf) • **Exercises 1-5**
Learning objectives	By the end of the lesson: • Students should be able to determine the dimensions of the given matrix and the required element and to solve all the exercises on their own.

Students should have no problem learning this lesson. The teacher should possibly emphasize or repeat some details several times to make them easier for students to remember. This will make it easier for students to solve the exercises on their own.

Lesson 2: Matrix arithmetic

Resources	• **Whiteboard** • Lesson 2 https://maremathics.pfst.hr/wp-content/uploads/2021/09/IO2-3-Matrices-and-Determinants-2.pdf • MareMathics_SummerSchool_Split_Matrices • Solving examples 4 and 7 using Geogebra • https://maremathics.pfst.hr/index.php/2021/09/02/3-matrices-and-determinants-2/#matrix-intro • https://maremathics.pfst.hr/index.php/2021/09/02/3-matrices-and-determinants-2/#matrix-multi-row-col • https://maremathics.pfst.hr/index.php/2021/09/02/3-matrices-and-determinants-2/#matrix-multi-col-row • https://maremathics.pfst.hr/index.php/2021/09/02/3-matrices-and-determinants-2/#matrix-multi-ex3 • https://maremathics.pfst.hr/index.php/2021/09/02/3-matrices-and-determinants-2/#appli-matrix-multi • https://maremathics.pfst.hr/index.php/2021/09/02/3-matrices-and-determinants-2/#appli-matrix-multi-part1 • https://maremathics.pfst.hr/index.php/2021/09/02/3-matrices-and-determinants-2/#appli-matrix-multi-part2 • **Exercises 6-10** • **Generating_a_matrix_polynomial – Matlab tool primarily intended for teachers**

Learning objectives	• To learn how to add, subtract, multiply and potentiate matrices

SUGGESTED TEACHING STRATEGIES, INPUT AND RESOURCES

The teacher should possibly emphasize or repeat some details several times to make them easier for students to remember. This will make it easier for students to solve the exercises on their own. Videos can help with self-study because the teacher is unlikely to be able to show all of them to students in the classroom. The teacher can divide the students into groups so that each group solves one task from the exercises.

 The Matlab tool can help teachers compose exam assignments.

Lesson 3: Determinant of a square matrix

Resources	• Whiteboard • Lesson 3 https://maremathics.pfst.hr/wp-content/uploads/2021/09/IO2-3-Matrices-and-Determinants-3.pdf • Exercises 11-12 • MareMathics Geogebra book (solution of the 11th exercise), https://www.geogebra.org/m/q3gg4jef • Determinant, Generating the determinant of the desired value – Excel VBA tools for teachers and students
Learning objectives	By the end of the lesson: students should learn how to calculate the determinant of a square matrix of any order.

SUGGESTED TEACHING STRATEGIES, INPUT AND RESOURCES

It is very important that students remember the Laplace expansion and then the most important properties of the determinant in order to understand the process of calculating the determinant. If students have learning difficulties, the teacher can solve some more examples.

It would be desirable for students to learn how to determine the determinant using GeoGebra.

Excel's VBA tools can be used to test solutions and compose new tasks.

Lesson 4: Inverse matrix

Resources	• Whiteboard • Lesson 4 https://maremathics.pfst.hr/wp-content/uploads/2021/09/IO2-3-Matrices-and-Determinants-4.pdf • Exercises 13-14 • Geogebra (solution of 13th exercise) • Inverse matrix (using Gauss-Jordan method) – Excel VBA tool for teachers and students
Learning objectives	To learn: • how the inverse matrix is determined by calculating determinants • Gauss-Jordan method procedure

SUGGESTED TEACHING STRATEGIES, INPUT AND RESOURCES

Every student must learn the Gaussian and Gauss-Jordan method. The videos in the Lesson 6 can help. If necessary, the teacher should solve additional examples. The solution can be tested with an Excel tool.

Lesson 5: Matrix equations

Resources	• **Whiteboard** • Lesson 5 https://maremathics.pfst.hr/wp-content/uploads/2021/09/IO2-3-Matrices-and-Determinants-5.pdf • Exercises 15-20
Learning objectives	By the end of the lesson: • Students should remember that matrix multiplication is associative but not commutative • *all* students should know how to solve simpler matrix equations.

At the lecture, matrix arithmetic is first briefly repeated. After listening to the lecture, the students study the pdf file once again. The teacher then divides them into groups. Each group solves several tasks from the exercises. Finally, tasks are compared and controlled.

Lesson 6: Matrix rank

Resources	• **Whiteboard** • Lesson 6 https://maremathics.pfst.hr/wp-content/uploads/2021/09/IO2-3-Matrices-and-Determinants-6.pdf • https://maremathics.pfst.hr/index.php/2021/09/02/3-matrices-and-determinants-2/#gauss-intro-task-1 • https://maremathics.pfst.hr/index.php/2021/09/02/3-matrices-and-determinants-2/#gauss-intro-task-2 • https://maremathics.pfst.hr/index.php/2021/09/02/3-matrices-and-determinants-2/#gauss-intro-task-3 • https://maremathics.pfst.hr/index.php/2021/09/02/3-matrices-and-determinants-2/#gauss-method-ex1 • https://maremathics.pfst.hr/index.php/2021/09/02/3-matrices-and-determinants-2/#gauss-method-ex2 • https://maremathics.pfst.hr/index.php/2021/09/02/3-matrices-and-determinants-2/#gauss-method-ex3 • Exercise 21 • **Generating_a_matrix_of_the_desired_rank – Matlab tool primarily intended for teachers**
Learning objectives	• To learn the procedure of the Gaussian method • To know how to determine the rank of any matrix using the Gaussian method

The students should watch all videos the day before listening to Lesson 6. Then they listen to the lecture, read the pdf file and finally solve Exercise 21. Matlab's tool can be used in compiling tasks in which the rank of the matrix should be determined.

Lesson 7: Systems of linear algebraic equations

Resources	• **Whiteboard** • **MareMathics_SummerSchool_Split_Matrices.pptx** • **Lesson 7 https://maremathics.pfst.hr/wp-content/uploads/2021/09/IO2-3-Matrices-and-Determinants-7.pdf** • **Exercises 22-27** • **Solving_a_linear_system – Matlab tool for teachers and students** • **Solving a system with a unique solution in which m=n=r (Gauss-Jordan) - Excel VBA tool for teachers and students**

	• **Generating_a_system_with_a_unique_solution – Matlab tool primarily intended for teachers**
	• **Generating_a_system_with_an_infinite_number_of_solutions – Matlab tool primarily intended for teachers**
	• **Generating_an_impossible_system – Matlab tool primarily intended for teachers**
Learning objectives	• To remember when a linear system has at least one solution and how to find them in that case

Start with an example of applying a rank matrix on page 15 of a powerpoint presentation. Continue with the pdf file. Students read this file and then solve Exercises 22-27 on their own. Additional tools are used to solve assigned tasks faster or to assign and solve new tasks.

Presentation Screens:

Lesson 8: Some examples of matrix applications

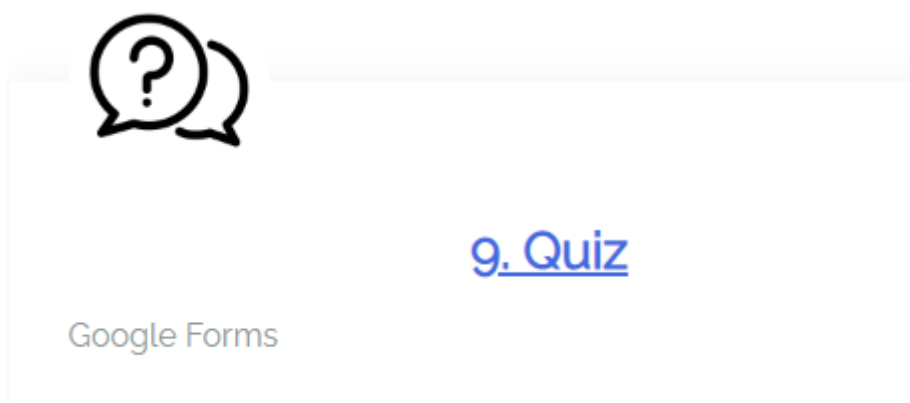| Resources	• **Whiteboard** • **Lesson 8 https://maremathics.pfst.hr/wp-content/uploads/2021/09/IO2-3-Matrices-and-Determinants-8.pdf**
Learning objectives	• To remember some more interesting applications of matrices • To make at least one applet in Geogebra or one Matlab program with the application of matrices

Students read a pdf file the day before the lecture. At the lecture, the teacher discusses with the students the applications of the matrices listed in the document. Eventually, some new applications are mentioned. For homework, students must, with the help of teacher, create several programs in Geogebra or Matlab with the application of matrices. Videos

https://maremathics.pfst.hr/index.php/2021/09/02/3-matrices-and-determinants-2/#appli-matrix-multi-part1
https://maremathics.pfst.hr/index.php/2021/09/02/3-matrices-and-determinants-2/#appli-matrix-multi-part2

could be helpful.

QUIZ

9. Quiz

Google Forms

If there is a problem with opening this quiz, an user can copy and run the following link in Crom or FireFox browser.

https://forms.gle/Fk5e5PQVFLAXyjih7