



## Teacher's Manual

# VECTORS

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## MareMathics

Innovative Approach in Mathematical Education for Maritime  
Students

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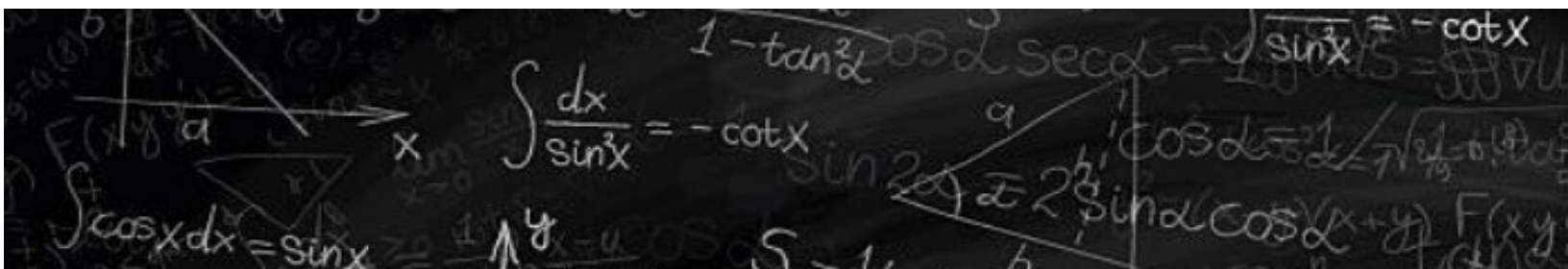
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<https://maremathics.pfst.hr/>

# Manual for teachers

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Reviewed by Anita Gudelj, Croatia





*The Manual is the outcome of the collaborative work of all the Partners for the development of the MareMathics Project.*

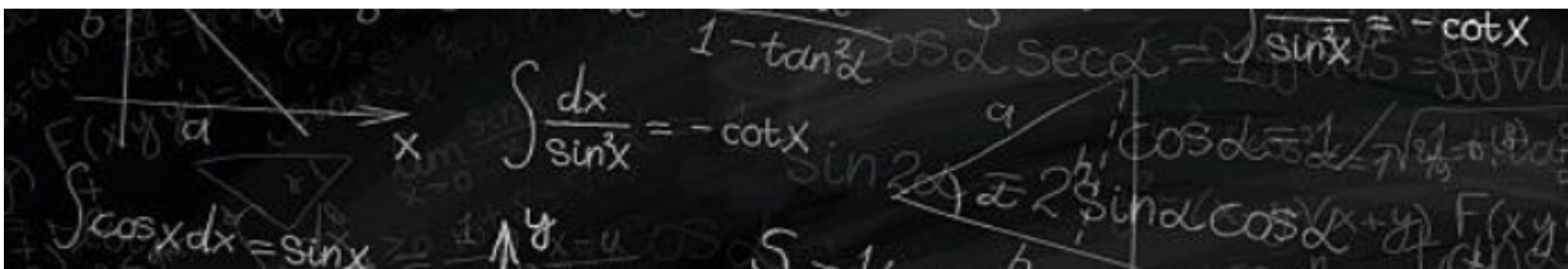
**Partners in the project:**



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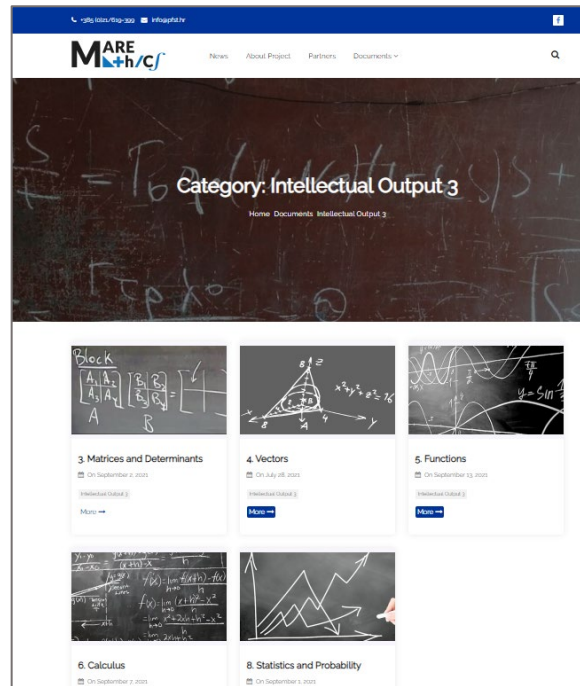
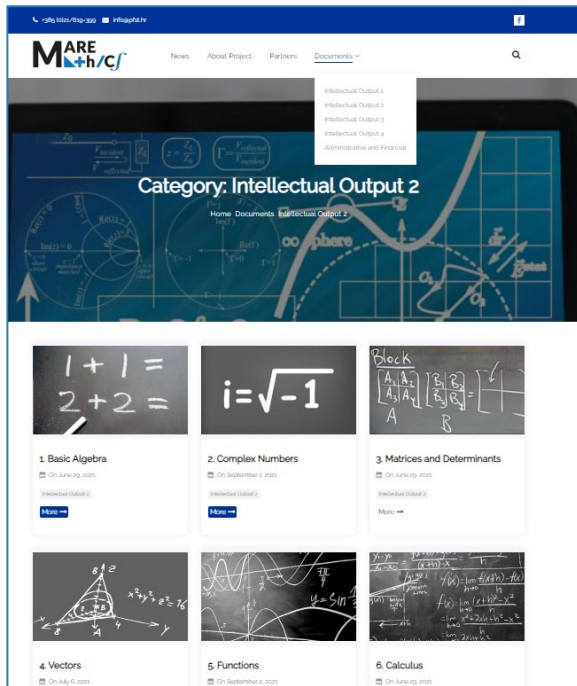
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## *VECTORS: Teaching and Learning PLAN*

The goal of this material and related resources is to assist teachers in planning their lessons allowing achieving learning outcomes posted in the course's syllabus. It enables teachers to design student activities to encourage students to learn.

The resources are picked from project *MareMathics* and available on the <https://maremathics.pfst.hr/>.



Name of Unit <i>Vectors</i>	Workload Lecture: 90 min Exercises: 90 min	Handbook Unit 4. Vectors
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## Introduction

The unit Vectors begins with the definition of the concept of vector and its geometrical depiction, and then the basic operations including vectors are defined. Scalar, vector and mixed product as well as their applications are presented.

Knowledge of vectors is important in developing students' understanding of the representation and behaviour of an object in the plane  $\mathbb{R}^2$  and space  $\mathbb{R}^3$  using different notations, and the ability to effectively apply these notations to explore the geometry of a situation. Vectors are used in many maritime fields, including navigation and engineering structural analysis which can be seen from the last part.

**AIM:** To acquire skills in solving tasks with vectors, and also to understand concepts standing behind those calculations. Vectors are also important for a number of real-life applications in the maritime field.

### Learning outcomes

At the end of this lecture, students should be able to

1. Understand that a vector is a quantity that has a magnitude and direction
2. Graphically represent a vector using a scaled diagram
3. Determine equal and opposite vectors, magnitude and unit vector
4. Express vector as linear combination of vectors
5. Demonstrate vectors in the coordinate system
6. Calculate with vectors algebraically and graphically
  - To add vectors using a head-to-tail addition method and a scaled vector addition diagram and be able to identify the magnitude and direction of the resultant
  - To add right-angle vectors, using the Pythagorean Theorem to determine the resultant magnitude and trigonometric functions to determine the resultant direction
  - To resolve vectors into components and to use the component method in order to add two or more non-perpendicular vectors in order to determine the resultant
7. Apply scalar, vector and mixed vector product
8. Chooses and uses appropriate technology to solve problems in a range of contexts.



**Key words of this Unit:**

Vector, Component form, Scalar product, Vector product, Mixed product

**Previous knowledge of mathematics:** Content from trigonometry, determinants, geometry in the plane  $\mathbb{R}^2$  and space  $\mathbb{R}^3$

**Relatedness with solving problems in the maritime field:** The real-world problems encountered most often with vectors are navigation problems. These navigation problems use variables like speed or velocity of vessel and direction or course to form vectors for computation. Some navigation problems ask us to find the actual (ground) speed of a vessel in wind situation using the combined forces of the wind and the vessel's velocity. Additionally, problems such as plotting, effect of ocean current on navigation, Cremona diagram used in statics of trusses to determine the forces in members can be solved by vector algebra.

**Contents:****4 VECTORS**

- 4.1 The concept of vector
- 4.2 Three basic vector operations
- 4.3 Scalar, vector and mixed triple products
- 4.4 Vectors in rectangular coordinate system
- 4.5 Performing Operations in Component Form
- 4.6 Exercises
- 4.7 Connectedness and application in the maritime field

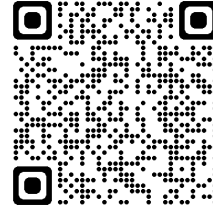
**Assessment strategies:**

Evaluating students activity during lesson.

**MareMathics Teacher Toolkit and Digital Resources:**

- Videos
- Geogebra
- Quizzes
- These resources are for revision at the end of the lesson. They could also be used to introduce some of the concepts, but you may need to split the work into sections and provide extra examples and problems.
  - You will be quizzed on how to find the unit vector, characteristics of vectors and how they relate to each other, and how to find the components of a vector.





Useful websites: [Vectors](#) | [Precalculus](#) | [Math](#) | [Khan Academy](#)





## LESSON FLOW

Time	Sequence	Content	Teacher activities	Student activities	Points for discussion
10 min 10 min	Starter/Introduction Presentation 4.1 Exercise 4.1	The concept of vector	Frontal then questioning Group work	Active listening and contributing to questions Solving exercise	Are students able to recognize the difference between scalars and vectors?
10 min 10 min	Presentation 4.2 Task 3.1	Three basic vector operations	Frontal and questioning Group work	Active listening and contributing to questions Solving exercise	
15 min	Presentation 4.3	Scalar, vector and mixed triple products	Frontal Explains task and supports (videos, GeoGebra files)	Active listening and contributing to questions	
20 min 40 min	Presentation 4.4 Task 3.2, 3.3, 3.4, 3.5, 3.6	Vectors in rectangular coordinate system	Frontal Discussion using solved examples	Active listening Solving exercise	Are students able to demonstrate vectors in the coordinate system?
15 min 30 min	Presentation 4.5 Task 3.7, 3.8 and 3.9	Performing Operations in Component Form	Frontal Discussion using solved examples	Active listening Discussion Solving exercise	Apply scalar, vector and mixed vector product.
20 min	Presentation 4.7	Connectedness and application in the maritime field	Frontal Discussion using solved examples	Active listening Discussion	Can we apply vectors in real life?



## SUGGESTED TEACHING STRATEGIES, INPUT AND RESOURCES

### Lesson 1: The concept of vector

	<ul style="list-style-type: none"> <li>• Whiteboard</li> <li>• Lesson 1 <a href="https://maremathics.pfst.hr/wp-content/uploads/2021/07/IO2-4-Vectors-1.pdf">https://maremathics.pfst.hr/wp-content/uploads/2021/07/IO2-4-Vectors-1.pdf</a> <ul style="list-style-type: none"> <li>• <a href="https://maremathics.pfst.hr/index.php/2021/07/28/4-vectors-2/#vectors-intro">https://maremathics.pfst.hr/index.php/2021/07/28/4-vectors-2/#vectors-intro</a></li> </ul> </li> <li>• Exercise 4.1           <ul style="list-style-type: none"> <li>• Using GeoGebra teachers can show to students how to use vectors. <a href="https://www.geogebra.org/m/efjh9bmr">https://www.geogebra.org/m/efjh9bmr</a></li> <li>• <a href="https://www.geogebra.org/m/bmhj5jtu">https://www.geogebra.org/m/bmhj5jtu</a> Quiz for students</li> </ul> </li> </ul>
<b>Learning objectives</b>	<p>By the end of the lesson:</p> <p>Students should be able to identify and use vector notation and understand the difference between vector and a scalar quantity. Students should have no problem learning this lesson.</p>

## Lesson 2: Three Basic Vector Operations

	<ul style="list-style-type: none"> <li>• Whiteboard</li> <li>• Lesson 2 <a href="https://maremathics.pfst.hr/wp-content/uploads/2021/07/IO2-4-Vectors-2.pdf">https://maremathics.pfst.hr/wp-content/uploads/2021/07/IO2-4-Vectors-2.pdf</a> <ul style="list-style-type: none"> <li>• Teachers can show students how to add vectors <a href="https://www.geogebra.org/m/aehtatkf#material/fuvcuujw">https://www.geogebra.org/m/aehtatkf#material/fuvcuujw</a></li> </ul> </li> <li>• Task 3.1</li> </ul>
<b>Learning objectives</b>	<p>By the end of the lesson:</p> <p style="padding-left: 40px;">Students should be able to recognize the difference between scalars and vectors and to use triangle method to add vectors.</p>

The teacher should possibly emphasize or repeat some details several times to make them easier for students to remember. Geogebra can help with self-study.



## Lesson 3 Scalar, vector and mixed triple product

	<ul style="list-style-type: none"> <li>• Whiteboard</li> <li>• Lesson 3 <a href="https://maremathics.pfst.hr/wp-content/uploads/2021/07/IO2-4-Vectors-3.pdf">https://maremathics.pfst.hr/wp-content/uploads/2021/07/IO2-4-Vectors-3.pdf</a></li> </ul>
<b>Learning objectives</b>	<p>By the end of the lesson:</p> <p>Students should be able to recognize the difference between scalar, vector and mixed products.</p>



## Lesson 4 Vectors in rectangular coordinate system

	<ul style="list-style-type: none"> <li>• Whiteboard</li> <li>• Lesson 4 <a href="https://maremathics.pfst.hr/wp-content/uploads/2021/07/IO2-4-Vectors-4.pdf">https://maremathics.pfst.hr/wp-content/uploads/2021/07/IO2-4-Vectors-4.pdf</a> <ul style="list-style-type: none"> <li>• Teachers can show to students: <a href="https://www.geogebra.org/m/aehtmatkf#material/kqgqgymz">https://www.geogebra.org/m/aehtmatkf#material/kqgqgymz</a></li> <li>• <a href="https://www.geogebra.org/m/aehtmatkf#material/xmachmar">https://www.geogebra.org/m/aehtmatkf#material/xmachmar</a></li> </ul> </li> <li>• Task 3.2 – 3.6</li> </ul>
<b>Learning objectives</b>	<p>By the end of the lesson: Students should be able to demonstrate vectors in the coordinate system and calculate with vectors algebraically and graphically.</p>

It is very important that students learn how to calculate with vectors algebraically. If students have learning difficulties the teacher can solve some more examples using Geogebra.

## Lesson 5 Performing Operations in Component Form

	<ul style="list-style-type: none"> <li>• Whiteboard</li> <li>• Lesson 5 <a href="https://maremathics.pfst.hr/wp-content/uploads/2021/07/IO2-4-Vectors-5.pdf">https://maremathics.pfst.hr/wp-content/uploads/2021/07/IO2-4-Vectors-5.pdf</a> <ul style="list-style-type: none"> <li>• <a href="https://maremathics.pfst.hr/index.php/2021/07/28/4-vectors-2/#cross-vector-product">https://maremathics.pfst.hr/index.php/2021/07/28/4-vectors-2/#cross-vector-product</a></li> <li>• <a href="https://maremathics.pfst.hr/index.php/2021/07/28/4-vectors-2/#vectors-ex2">https://maremathics.pfst.hr/index.php/2021/07/28/4-vectors-2/#vectors-ex2</a></li> <li>• <a href="https://maremathics.pfst.hr/index.php/2021/07/28/4-vectors-2/#vectors-ex3">https://maremathics.pfst.hr/index.php/2021/07/28/4-vectors-2/#vectors-ex3</a></li> </ul> </li> <li>• Task 3.7 – 3.9</li> </ul>
<b>Learning objectives</b>	<p>By the end of the lesson: Students should be able to apply scalar, vector and mixed product.</p>



## Lesson 7 Connectedness and application in the maritime field

	<ul style="list-style-type: none"> <li>• Whiteboard</li> <li>• Lesson 7 <a href="https://maremathics.pfst.hr/wp-content/uploads/2021/07/IO2-4-Vectors-7.pdf">https://maremathics.pfst.hr/wp-content/uploads/2021/07/IO2-4-Vectors-7.pdf</a> <ul style="list-style-type: none"> <li>• Teachers show to students :           <ul style="list-style-type: none"> <li><a href="https://www.geogebra.org/m/aehtatkf#material/g4r3pm7d">https://www.geogebra.org/m/aehtatkf#material/g4r3pm7d</a></li> <li><a href="https://www.geogebra.org/m/aehtatkf#material/fez2uvrm">https://www.geogebra.org/m/aehtatkf#material/fez2uvrm</a></li> <li><a href="https://www.geogebra.org/m/aehtatkf#material/dduywkxg">https://www.geogebra.org/m/aehtatkf#material/dduywkxg</a></li> </ul> </li> </ul> </li> </ul>
<b>Learning objectives</b>	<p>By the end of the lesson:</p> <p style="padding-left: 40px;">Students should be able to apply vectors in some example in the maritime field.</p>

Students should test their knowledge by solving an online quiz on the link:

<https://quizizz.com/join/quiz/623de43c26ea03001d0e0fc2/start?studentShare=true>

